Chapter 7

SETS AND PROBABILITY

7.1 Sets

Your Turn 1

There are three states whose names begin with O, Ohio and Oklahoma. Thus $\{x \mid x$ is a state that begins with the letter O$\} = \{\text{Ohio, Oklahoma, Oregon}\}$.

Your Turn 2

$\{2,4,6\} \subseteq \{6,2,4\}$ is true because each element of the first set is an element of the second set. (In this example the sets are in fact equal.)

Your Turn 3

A set of k distinct elements has 2^k subsets, so since there are four seasons, $\{x \mid x$ is a season of the year$\}$ has $2^4 = 16$ subsets.

Your Turn 4

$$U = \{0,1,2,...,10\}$$
$$A = \{3,6,9\}$$
$$B = \{2,4,6,8\}$$

Then $A' = \{0,1,2,4,5,7,8,10\}$. The elements common to A' and B are 2,4, and 8. Thus $A' \cap B = \{2,4,8\}$.

Your Turn 5

$$U = \{0,1,2,...,12\}$$
$$A = \{1,3,5,7,9,11\}$$
$$B = \{3,6,9,12\}$$
$$C = \{1,2,3,4,5\}$$

To find $A \cup (B \cap C')$ begin with the expression in parentheses and find C', which includes all the elements of the universal set that are not in C.

$$C' = \{6,7,8,9,10,11,12\}$$

The elements in C' that are also in B are 6, 9, and 12, so $B \cap C' = \{6,9,12\}$.

Now we list the elements of A and include any elements of $B \cap C'$ that are not already listed:

$$A \cup (B \cap C') = \{1,3,5,6,7,9,11,12\}$$

7.1 Exercises

1. $3 \in \{2,5,7,9,10\}$

The number 3 is not an element of the set, so the statement is false.

2. $6 \in \{-2,6,9,5\}$

Since 6 is an element of the given set, the statement is true.

3. $9 \notin \{2,1,5,8\}$

Since 9 is not an element of the set, the statement is true.

4. $3 \notin \{7,6,5,4\}$

Since 3 is not an element of the given set, the statement is true.

5. $\{2,5,8,9\} = \{2,5,9,8\}$

The sets contain exactly the same elements, so they are equal. The statement is true.

6. $\{3,7,12,14\} = \{3,7,12,14,0\}$

Two sets are equal only if they contain exactly the same elements. Since 0 is an element of the second set but not the first, the statement is false.

7. {All whole numbers greater than 7 and less than 10} $= \{8,9\}$

Since 8 and 9 are the only such numbers, the statement is true.

8. $\{x \mid x$ is an odd integer, $6 \le x \le 18\} = \{7,9,11,15,17\}$

The number 13 should be included in the set so the statement is false.

9. $0 \in \emptyset$

The empty set has no elements. The statement is false.

10. $\emptyset \in \{\emptyset\}$

Since $\{\emptyset\}$ has $\emptyset$ as a member, this statement is true.

In Exercises 11–22,

$$A = \{2,4,6,8,10,12\},$$
$$B = \{2,4,8,10\},$$
$$C = \{4,8,12\},$$
$$D = \{2,10\},$$
$$E = \{6\},$$
and $$U = \{2,4,6,8,10,12,14\}.$$

11. Since every element of A is also an element of U, A is a subset of U, written $A \subseteq U$.

12. Since every element of E is also an element of A, E is a subset of A, written $E \subseteq A$.

13. A contains elements that do not belong to E, namely 2, 4, 8, 10, and 12, so A is not a subset of E, written $A \not\subseteq E$.

14. Since 10 is an element of B but is not an element of C, B is not a subset of C, written $B \not\subseteq C$.

15. The empty set is a subset of every set, so $\emptyset \subseteq A$.

16. Since 0 is an element of $\{0,2\}$, but is not an element of D, $\{0,2\} \not\subseteq D$.

17. Every element of D is also an element of B, so D is a subset of B, $D \subseteq B$.

18. Since 2, 6, and 10 are elements of A, but are not elements of C, $A \not\subseteq C$.

19. Since every element of A is also an element of U, and $A \neq U$, $A \boxed{\subset} U$.

Since every element of E is also an element of A, and $E \neq A$, $E \boxed{\subset} A$.

Since every element of A is not also an element of E, $A \boxed{\not\subset} E$.

Since every element of B is not also an element of C, $B \boxed{\not\subset} C$.

Since $\emptyset$ is a subset of every set, and $\emptyset \neq A$, $\emptyset \boxed{\subset} A$.

Since every element of $\{0,2\}$ is not also an element of D, $\{0,2\} \boxed{\not\subset} D$.

Since every element of D is not also an element of B, and $D \neq B$, $D \boxed{\subset} B$.

Since every element of A is not also an element of C, $A \boxed{\not\subset} C$.

21. A set with n distinct elements has 2^n subsets. A has $n = 6$ elements, so there are exactly $2^6 = 64$ subsets of A.

22. Since B has 4 elements, it has 2^4 or 16 subsets. There are exactly 16 subsets of B.

23. A set with n distinct elements has 2^n subsets, and C has $n = 3$ elements. Therefore, there are exactly $2^3 = 8$ subsets of C.

24. Since D has 2 elements, it has 2^2 or 4 subsets. There are exactly 4 subsets of D.

25. Since $\{7,9\}$ is the set of elements belonging to both sets, which is the intersection of the two sets, we write

$$\{5,7,9,19\} \cap \{7,9,11,15\} = \{7,9\}.$$

26. $\{8,11,15\} \cap \{8,11,19,20\} = \{8,11\}$

$\{8,11\}$ is the set of all elements belonging to both of the first two sets, so it is the intersection of those sets.

27. Since $\{1,2,5,7,9\}$ is the set of elements belonging to one or the other (or both) of the listed sets, it is their union.

$$\{2,1,7\} \cup \{1,5,9\} = \{1,2,5,7,9\}$$

28. Since $\{6,12,14,16,19\}$ is the set of elements belonging to one or the other (or both) of the listed sets, it is their union.

$$\{6,12,14,16\} \cup \{6,14,19\} = \{6,12,14,16,19\}$$

29. Since $\emptyset$ contains no elements, there are no elements belonging to both sets. Thus, the intersection is the empty set, and we write

$$\{3,5,9,10\} \cap \emptyset = \emptyset.$$

30. $\{3,5,9,10\} \cup \emptyset = \{3,5,9,10\}$

The empty set contains no elements, so the union of any set with the empty set will result in an answer set that is identical to the original set. (On the other hand, $\{3,5,9,10\} \cap \emptyset = \emptyset$.)

31. $\{1,2,4\}$ is the set of elements belonging to both sets, and $\{1,2,4\}$ is also the set of elements in the first set or in the second set or possibly both. Thus,

$$\{1,2,4\} \cap \{1,2,4\} = \{1,2,4\}$$

and

$$\{1,2,4\} \cup \{1,2,4\} = \{1,2,4\}$$

are both true statements.

32. $\{0,10\}$ is the set of elements belonging to both of the two sets on the left, and it is also the set of elements belonging to one or the other of these two sets, or to both. Thus

$$\{0,10\} \cap \{10,0\} = \{0,10\}$$

and

$$\{0,10\} \cup \{10,0\} = \{0,10\}$$

are both true statements.

In Exercises 35–44,

$$\begin{aligned} U &= \{1,2,3,4,5,6,7,8,9\}, \\ X &= \{2,4,6,8\}, \\ Y &= \{2,3,4,5,6\}, \\ \text{and} \quad Z &= \{1,2,3,8,9\}. \end{aligned}$$

35. $X \cap Y$, the intersection of X and Y, is the set of elements belonging to both X and Y. Thus,

$$\begin{aligned} X \cap Y &= \{2,4,6,8\} \cap \{2,3,4,5,6\} \\ &= \{2,4,6\}. \end{aligned}$$

36. $X \cup Y$, the union of X and Y, is the set of all elements belonging to X or Y or both. Thus,

$$X \cup Y = \{2,3,4,5,6,8\}.$$

37. X', the complement of X, consists of those elements of U that are not in X. Thus,

$$X' = \{1,3,5,7,9\}.$$

38. Y', the complement of Y, is the set of all elements of U that do not belong to Y. Thus,

$$Y' = \{1,7,8,9\}.$$

39. From Exercise 37, $X' = \{1,3,5,7,9\}$; from Exercise 38, $Y' = \{1,7,8,9\}$. There are no elements common to both X' and Y' so

$$X' \cap Y' = \{1,7,9\}.$$

40. $$\begin{aligned} X' \cap Z &= \{1,3,5,7,9\} \cap \{1,2,3,8,9\} \\ &= \{1,3,9\} \end{aligned}$$

41. First find $X \cup Z$.

$$\begin{aligned} X \cup Z &= \{2,4,6,8\} \cup \{1,2,3,8,9\} \\ &= \{1,2,3,4,6,8,9\} \end{aligned}$$

Now find $Y \cap (X \cup Z)$.

$$\begin{aligned} Y \cap (X \cup Z) &= \{2,3,4,5,6\} \cap \{1,2,3,4,6,8,9\} \\ &= \{2,3,4,6\} \end{aligned}$$

42. From Exercise 37, $X' = \{1,3,5,7,9\}$; from Exercise 38, $Y' = \{1,7,8,9\}$.

$$\begin{aligned} X' \cap (Y' \cup Z) &= \{1,3,5,7,9\} \cap (\{1,7,8,9\} \cup \{1,2,3,8,9\}) \\ &= \{1,3,5,7,9\} \cap \{1,2,3,7,8,9\} \\ &= \{1,3,7,9\} \end{aligned}$$

43. $U = \{1,2,3,4,5,6,7,8,9\}$ and $Z = \{1,2,3,8,9\}$, so $Z' = \{4,5,6,7\}$.

From Exercise 38, $Y' = \{1,7,8,9\}$.

$$\begin{aligned} (X \cap Y') \cup (Z' \cap Y') &= (\{2,4,6,8\} \cap \{1,7,8,9\}) \\ &\quad \cup (\{4,5,6,7\} \cap \{1,7,8,9\}) \\ &= \{8\} \cup \{7\} \\ &= \{7,8\} \end{aligned}$$

44. From Exercise 37, $X' = \{1,3,5,7,9\}$.

$$\begin{aligned} (X \cap Y) \cup (X' \cap Z) &= (\{2,4,6,8\} \cap \{2,3,4,5,6\}) \\ &\quad \cup (\{1,3,5,7,9\} \cap \{1,2,3,8,9\}) \\ &= \{2,4,6\} \cup \{1,3,9\} \\ &= \{1,2,3,4,6,9\} \end{aligned}$$

45. $$\begin{aligned} (A \cap B) \cup (A \cap B') &= (\{3,6,9\} \cap \{2,4,6,8\}) \\ &\quad \cup (\{3,6,9\} \cap \{0,1,3,5,7,9,10\}) \\ &= \{6\} \cup \{3,9\} \\ &= \{3,6,9\} \\ &= A \end{aligned}$$

47. M' consists of all students in U who are not in M, so M' consists of all students in this school not taking this course.

48. $M \cup N$ is the set of all students in this school taking this course or taking accounting.

49. $N \cap P$ is the set of all students in this school taking both accounting and zoology.

50. $N' \cap P'$ is the set of all students in this school not taking accounting and not taking zoology.

51. $$\begin{aligned} A &= \{2,4,6,8,10,12\}, \\ B &= \{2,4,8,10\}, \\ C &= \{4,8,12\}, \\ D &= \{2,10\}, \\ E &= \{6\}, \\ U &= \{2,4,6,8,10,12,14\} \end{aligned}$$

A pair of sets is disjoint if the two sets have no elements in common. The pairs of these sets that are disjoint are B and E, C and E, D and E, and C and D.

52. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$X = \{2, 4, 6, 8\}$
$Y = \{2, 3, 4, 5, 6\}$
$Z = \{1, 2, 3, 8, 9\}$

Since each pair of sets has at least one element in common (for example, the element 2), none of the pairs of sets are disjoint.

53. B' is the set of all stocks on the list with a closing price below \$60 or above \$70.

$B' =$ {AT&T, Coca-Cola, FedEx, Disney}

54. $A \cap B$ is the set of all stocks on the list with a high price greater than \$50 and a closing price between \$60 and \$70.

$A \cap B =$ {McDonald's, PepsiCo}

55. $(A \cap B)'$ is the set of all stocks on the list that do not have both a high price greater than \$50 and a closing price between \$60 and \$70.

$(A \cap B)' =$ {AT&T, Coca-Cola, FedEx, Disney}

56. $(C \cup D)'$ is the set of all stocks on the list that do not have either a positive price change or a low price less than \$40.

$$(C \cup D)' = \emptyset$$

57. $A = \{1, 2, 3, \{3\}, \{1, 4, 7\}\}$

(a) $1 \in A$ is true.

(b) $\{3\} \in A$ is true.

(c) $\{2\} \in A$ is false. $(\{2\} \subseteq A)$

(d) $4 \in A$ is false. $(4 \in \{1, 4, 7\})$

(e) $\{\{3\}\} \subset A$ is true.

(f) $\{1,4,7\} \in A$ is true.

(g) $\{1, 4, 7\} \subseteq A$ is false. $(\{1, 4, 7\} \in A)$

58. $B = \{a, b, c, \{d\}, \{e, f\}\}$

(a) $a \in B$ is true.

(b) $\{b, c, d\} \subset B$ is false. $(d \notin B)$

(c) $\{d\} \in B$ is true.

(d) $\{d\} \subseteq B$ is false. $(\{d\} \in B)$

(e) $\{e, f\} \in B$ is true.

(f) $\{a, \{e, f\}\} \subset B$ is true.

(g) $\{e, f\} \subset B$ is false. $(\{e, f\} \in B)$

For Exercises 59 through 62 refer to this abbreviated version of the table:

Vanguard 500 (V)	**Fidelity New Millennium Fund (F)**	**Janus Perkins Large Cap Value (J)**	**Templeton Large Cap Value Fund (T)**
Exxon	Pfizer	Exxon	IBM
Apple	Cisco	GE	GE
GE	Wal-Mart	Wal-Mart	HP
IBM	Apple	AT&T	Home Depot
JPMorgan	JPMorgan	JPMorgan	Aflac

59. $V \cap J =$ {Exxon, Apple, GE, IBM, JPMorgan}
$\cap$ {Exxon, GE, Wal-Mart, AT&T, JPMorgan}
$=$ {Exxon, GE, JPMorgan}

60.
$V \cap (F \cup T) =$ {Exxon, Apple, GE, IBM, JPMorgan}
$\cap$ ({Pfizer, Cisco, Wal-Mart, Apple, JPMorgan}
$\cup$ {IBM, GE, HP, Home Depot, Aflac})
$=$ {Exxon, Apple, GE, IBM, JPMorgan}
$\cap$ {Pfizer, Cisco, Wal-Mart, Apple, JPMorgan, IBM, GE, HP, Home Depot, Aflac}
$=$ {Apple, GE, IBM, JPMorgan}

61.
$J \cup F =$ {Exxon, GE, Wal-Mart, AT&T, JPMorgan}
$\cup$ {Pfizer, Cisco,Wal-Mart, Apple, JPMorgan}
$=$ {Exxon, GE, Wal-Mart, AT&T, JPMorgan, Pfizer, Cisco, Apple}
$(J \cup F)' =$ {IBM, HP, Home Depot, Aflac}

62. $J' \cap T'$ is the set of elements neither in J nor in T. This is the set of elements in the first two columns of the table that are not in the last two columns.
$J' \cap T' =$ {Pfizer, Cisco, Apple}

63. The number of subsets of a set with k elements is 2^k, so the number of possible sets of customers (including the empty set) is $2^9 = 512$.

64. $U = \{s, d, c, g, i, m, h\}$ and $O = \{i, m, h, g\}$, so $O' = \{s, d, c\}$.

65. $U = \{s, d, c, g, i, m, h\}$ and $N = \{s, d, c, g\}$, so

$$N' = \{i, m, h\}.$$

66. $N \cap O = \{s, d, c, g\} \cap \{i, m, h, g\} = \{g\}$

67. $N \cup O = \{s, d, c, g\} \cup \{i, m, h, g\}$
$= \{s, d, c, g, i, m, h\} = U$

68. $N \cap O' = \{s, d, c, g\} \cap \{s, d, c\} = \{s, d, c\}$

69. The number of subsets of a set with 51 elements (50 states plus the District of Columbia) is

$$2^{51} \approx 2.252 \times 10^{15}.$$

70. The total number of subsets is $2^4 = 16$. The number of subsets with no elements is 1, and the number of subsets with one element is 4. Therefore, the number of subsets with at least two elements is $16 - (1 + 4)$ or 11.

For Exercises 71 through 76 refer to this table:

Network	Subscribers (millions)	Launch	Content
The Discovery Channel	98.0	1985	Nonfiction, nature, science
TNT	98.0	1988	Movies, sports, original programming
USA Network	97.5	1980	Sports, family entertainment
TLC	97.3	1980	Original programming, family entertainment
TBS	97.3	1976	Movies, sports, original programming

71. $F = \{$USA, TLC, TBS$\}$

72. $G = \{$TNT, USA, TBS$\}$

73. $H = \{$Discovery, TNT$\}$

74. $F \cap H = \{$USA, TLC, TBS$\} \cap \{$Discovery, TNT$\} = \emptyset$; the set of networks launched before 1985 that also have more than 97.6 million viewers.

75. $G \cup H = \{$TNT, USA, TBS$\} \cup \{$Discovery, TNT$\}$
$= \{$TNT, USA, TBS, Discovery$\}$;
the set of networks that feature sports or that have more than 97.6 million viewers.

76. $G' = \{$Discovery, TLC$\}$; the set of networks that do not feature sports.

77. Joe should always first choose the complement of what Dorothy chose. This will leave only two sets to choose from, and Joe will get the last choice.

78. **(a)** $A \cup (B \cap C)'$

$B \cap C$ is the set of states with both a population over 4,000,000 and an area greater than 40,000 square miles. Therefore, $(B \cap C)'$ is the set of states with not both a population over 4,000,000 and an area greater than 40,000 square miles. As a result, $A \cup (B \cap C)'$ is the set of states whose name contains the letter ***e*** or which are not both over 4,000,000 in population and over 40,000 square miles in area.

(b)

$(B \cap C)' = (\{$Alabama, Colorado, Florida, Indiana, Kentucky, New Jersey$\}$
$\cap \{$Alabama, Alaska, Colorado, Florida, Kentucky, Nebraska$\})'$
$= \{$Alabama, Colorado, Florida, Kentucky$\}'$
$= \{$Alaska, Hawaii, Indiana, Maine, Nebraska, New Jersey$\}$

$A \cup (B \cap C)' = \{$Kentucky, Maine, Nebraska, New Jersey$) \cup \{$Alaska, Hawaii, Indiana, Maine, Nebraska, New Jersey$\}$
$= \{$Alaska, Hawaii, Indiana, Kentucky, Maine, Nebraska, New Jersey$\}$

79. **(a)** $(A \cup B') \cap C$

$A \cup B$ is the set of states whose name contains the letter *e* or which have a population over 4,000,000. Therefore, $(A \cup B)'$ is the set of states which are not among those whose name contains the letter *e* or which have a population over 4,000,000. As a result, $(A \cup B)' \cap C$ is the set of states which are not among those whose name contains the letter *e* or which have a population over 4,000,000 and which also have an area over 40,000 square miles.

(b)

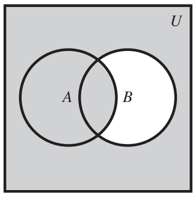
$(A \cup B)' = \{($Kentucky, Maine, Nebraska, New Jersey$\}$
$\cup \{$Alabama, Colorado, Florida, Indiana, Kentucky, New Jersey$\})'$
$= \{$Alabama, Colorado, Florida, Indiana, Kentucky, Maine, Nebraska, New Jersey$\}'$
$= \{$Alaska, Hawaii$\}$

$(A \cup B)' \cap C = \{$Alaska, Hawai$\} \cap \{$Alabama, Alaska, Colorado, Florida, Kentucky, Nebraska$\}$
$= \{$Alaska$\}$

7.2 Applications of Venn Diagrams

Your Turn 1

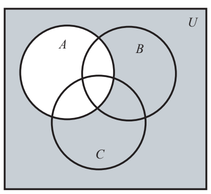
$A \cup B'$ is the set of elements in A or not in B or both in A and not in B.



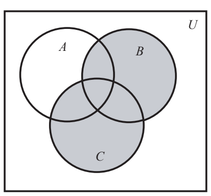
Your Turn 2

$A' \cap (B \cup C)$

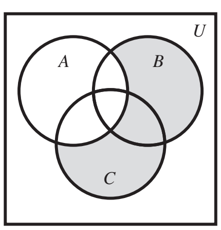
First find A'.



Then find $B \cup C$.



Then intersect these regions.

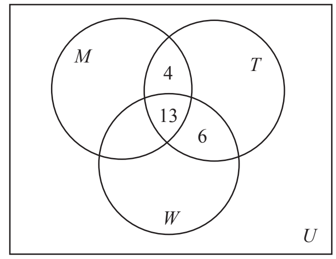


Your Turn 3

Start with $n(M \cap T \cap W) = 13$ and label the corresponding region with 13.

Since $n(M \cap T) = 17$, there are an additional 4 elements in $M \cap T$ but not in $M \cap T \cap W$. Label the corresponding region with 4.

Since $n(T \cap W) = 19$, there are an additional 6 elements in $T \cap W$ but not in $M \cap T \cap W$. Label the corresponding region with 6. The Venn diagram now looks like this:



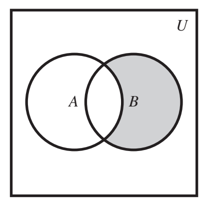
Since $n(T) = 46$, the number who ate only at Taco Bell is $46 - 4 - 13 - 6 = 23$.

Your Turn 4

Let T represent the set of those texting and M represent the set of those listening to music. The number in the lounge is $n(T \cup M) = n(T) + n(M) - n(T \cap M)$. We know that $n(T) = 15$, $n(M) = 11$, and $n(T \cap M) = 8$. Then $n(T \cup M) = 15 + 11 - 8 = 18$.

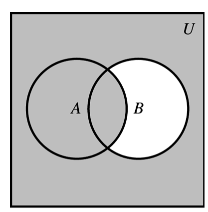
7.2 Exercises

1. $B \cap A'$ is the set of all elements in B *and* not in A.



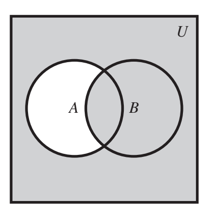
$B \cap A'$

2. $A \cup B'$ is the set of all elements in A *or* not in B, or both.



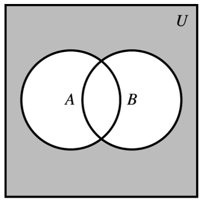
$A \cup B'$

3. $A' \cup B$ is the set of all elements that do not belong to A *or* that do belong to B, or both.



$A' \cup B$

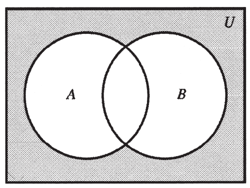
4. $A' \cap B'$ is the set of all elements not in A *and* not in B.



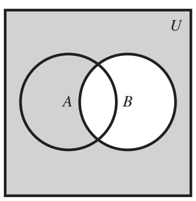
$A' \cap B'$

5. $B' \cup (A' \cap B')$

First find $A' \cap B'$, the set of elements not in A *and* not in B.



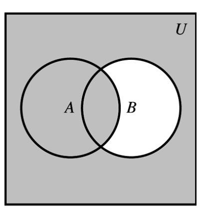
For the union, we want those elements in B' *or* $(A' \cap B')$, or both.



$B' \cup (A' \cap B')$

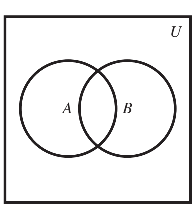
6. $(A \cap B) \cup B'$

First find $A \cap B$, the set of elements in A *and* in B. Now combine this region with B', the set of all elements not in B. For the union, we want those elements in $A \cap B$ *or* B', or both.



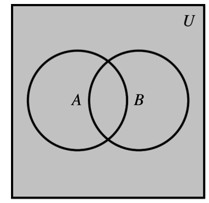
$(A \cap B) \cup B'$

7. U' is the empty set $\emptyset$.



$U' = \emptyset$

8. $\emptyset' = U$, so the entire region is shaded.

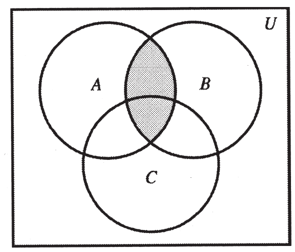


$\emptyset' = U$

9. Three sets divide the universal set into at most 8 regions. (Examples of this situation will be seen in Exercises 11–17.)

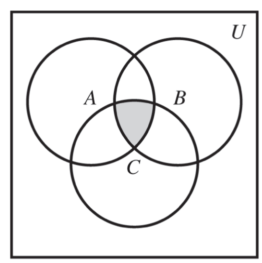
11. $(A \cap B) \cap C$

First form the intersection of A with B.



$A \cap B$

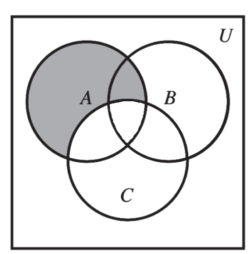
Now form the intersection of $A \cap B$ with C. The result will be the set of all elements that belong to all three sets.



$(A \cap B) \cap C$

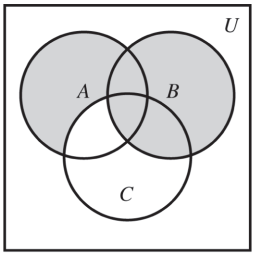
12. $(A \cap C') \cup B$

First find $A \cap C'$, the region in A *and* not in C.



$(A \cap C')$

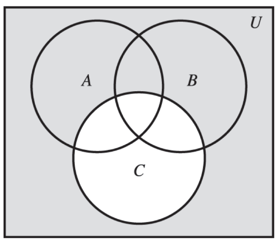
For the union, we want the region in $(A \cap C')$ *or* in B, or both.



$(A \cap C') \cup B$

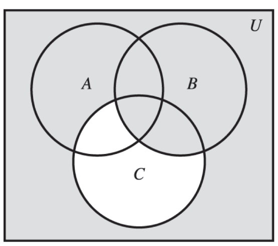
13. $A \cap (B \cup C')$

C' is the set of all elements in U that are not elements of C.



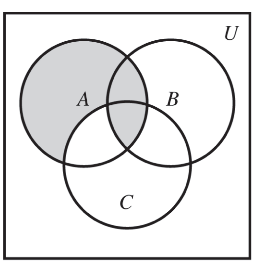
C'

Now form the union of C' with B.



$B \cup C'$

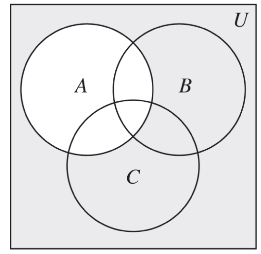
Finally, find the intersection of this region with A.



$A \cap (B \cup C')$

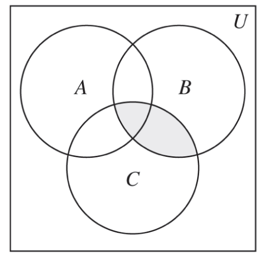
14. $A' \cap (B \cap C)$

First find A', the region not in A.



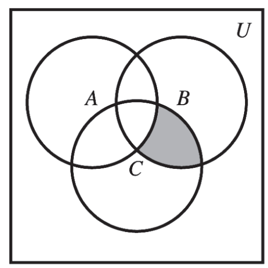
A'

Then find $B \cap C$, the region where B and C overlap.



$B \cap C$

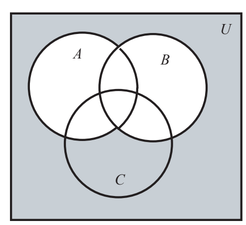
Now intersect these regions.



$A' \cap (B \cap C)$

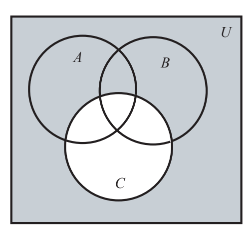
15. $(A' \cap B') \cap C'$

$A' \cap B'$ is the part of the universal set not in A *and* not in B:



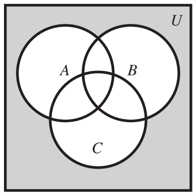
$A' \cap B'$

C' is the part of the universal set not in C:



C'

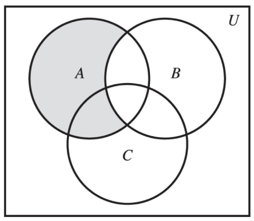
Now intersect the shaded regions in these two diagrams:



$(A' \cap B') \cap C'$

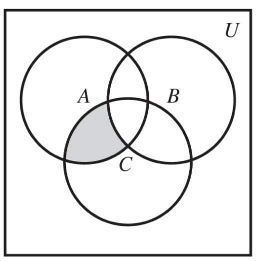
16. $(A \cap B') \cap C$

First find $A \cap B'$, the region in A *and* not in B.



$A \cap B'$

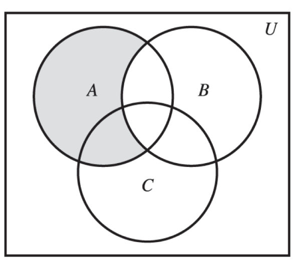
Now intersect this region with C.



$(A \cap B') \cap C$

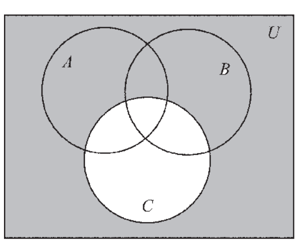
17. $(A \cap B') \cup C'$

First find $A \cap B'$, the region in A and not in B:



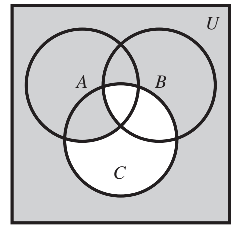
$A \cap B'$

C' is the region of the universal set not in C:



C'

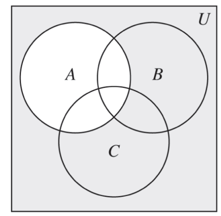
Now form the union of these two regions.



$(A \cap B') \cup C'$

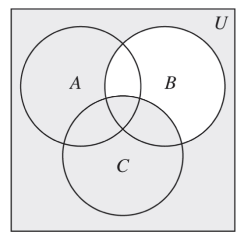
18. $A' \cap (B' \cup C)$

First find A'.



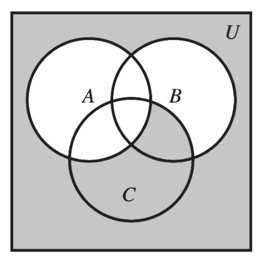
A'

Then find $B' \cup C$, the region not in B *or* in C, or both.



$B' \cup C$

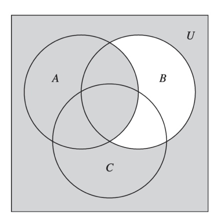
Now intersect these regions.



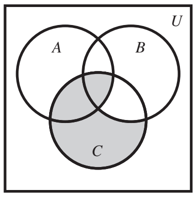
$A' \cap (B' \cup C)$

19. $(A \cup B') \cap C$

First find $A \cup B'$, the region in A or B' or both.



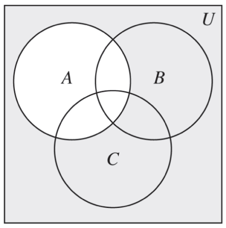
Intersect this with C.



$(A \cup B') \cap C$

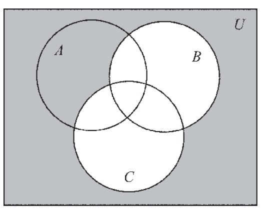
20. $A \cup (B' \cap C')$

First find A':



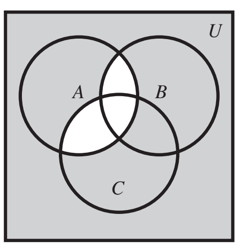
A'

Now find $B' \cap C'$:



$B' \cap C'$

Now find the union of these two regions:



$A' \cup (B' \cap C')$

21.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 5 + 12 - 4 \\ &= 13 \end{aligned}$$

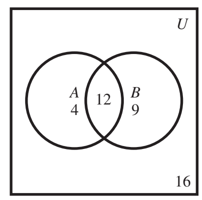
22.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 33 &= 15 + 30 - n(A \cap B) \\ 33 &= 45 - n(A \cap B) \\ n(A \cap B) &= 12 \end{aligned}$$

23.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 22 &= n(A) + 9 - 5 \\ 22 &= n(A) + 4 \\ 18 &= n(A) \end{aligned}$$

24.
$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 38 &= 13 + n(B) - 5 \\ 38 &= 8 + n(B) \\ n(B) &= 30 \end{aligned}$$

25. $n(U) = 41$
$n(A) = 16$
$n(A \cap B) = 12$
$n(B') = 20$

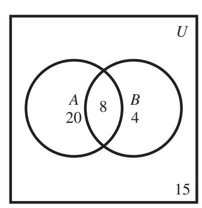
First put 12 in $A \cap B$. Since $n(A) = 16$, and 12 are in $A \cap B$, there must be 4 elements in A that are not in $A \cap B$. $n(B') = 20$, so there are 20 not in B. We already have 4 not in B (but in A), so there must be another 16 outside B *and* outside A. So far we have accounted for 32, and $n(U) = 41$, so 9 must be in B but not in any region yet identified. Thus $n(A' \cap B) = 9$.



26. $n(A) = 28$
$n(B) = 12$
$n(A \cup B) = 32$
$n(A') = 19$

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 32 &= 28 + 12 - n(A \cap B) \\ 32 &= 40 - n(A \cap B) \\ n(A \cap B) &= 8 \end{aligned}$$

To fill in the regions, start with $A \cap B$. $n(A) = 28$ and $n(A \cap B) = 8$, so $n(A \cap B') = 20$. $n(B) = 12$ and $n(A \cap B) = 8$, so $n(B \cap A') = 4$. Since $n(A') = 19, 4$ of which are accounted for in $B \cap A'$, 15 elements remain in $A' \cap B'$.



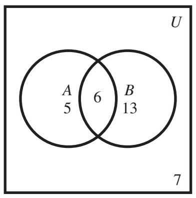
27. $n(A \cup B) = 24$

$n(A \cap B) = 6$

$n(A) = 11$

$n(A' \cup B') = 25$

Start with $n(A \cap B) = 6$. Since $n(A) = 11$, there must be 5 more in A not in B. $n(A \cup B) = 24$; we already have 11, so 13 more must be in B not yet counted. $A' \cup B'$ consists of all the region not in $A \cap B$, where we have 6. So far $5 + 13 = 18$ are in this region, so another $25 - 18 = 7$ must be outside both A and B.



28. $n(A') = 31$

$n(B) = 25$

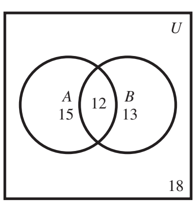
$n(A' \cup B') = 46$

$n(A \cap B) = 12$

$n(B) = 25$ and $n(A \cap B) = 12$, so $n(B \cap A') = 13$. Since $n(A') = 31$, of which 13 elements are accounted for, 18 elements are in $A' \cap B'$.

$$\begin{aligned} n(A' \cup B') &= n(A') + n(B') - n(A' \cap B') \\ 46 &= 31 + n(B') - 18 \\ 46 &= 13 + n(B') \\ 33 &= n(B') \end{aligned}$$

18 elements are in $A' \cap B'$, so the rest are in $A \cap B'$, and $n(A \cap B') = 15$.



29. $n(A) = 28$ $\quad n(B) = 34$ $\quad n(C) = 25$

$n(A \cap B) = 14$ $\quad n(B \cap C) = 15$ $\quad n(A \cap C) = 11$

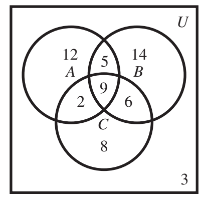
$n(A \cap B \cap C) = 9$

$n(U) = 59$

We start with $n(A \cap B \cap C) = 9$. If $n(A \cap B) = 14$, an additional 5 are in $A \cap B$ but not in $A \cap B \cap C$. Similarly, $n(B \cap C) = 15$, so $15 - 9 = 6$ are in $B \cap C$ but not in $A \cap B \cap C$. Also, $n(A \cap C) = 11$, so $11 - 9 = 2$ are in $A \cap C$ but not in $A \cap B \cap C$.

Now we turn our attention to $n(A) = 28$. So far we have $2 + 9 + 5 = 16$ in A; there must be another $28 - 16 = 12$ in A not yet counted. Similarly, $n(B) = 34$; we have $5 + 9 + 6 = 20$ so far, and $34 - 20 = 14$ more must be put in B.

For C, $n(C) = 25$; we have $2 + 9 + 6 = 17$ counted so far. Then there must be 8 more in C not yet counted. The count now stands at 56, and $n(U) = 59$, so 3 must be outside the three sets.



30. $n(A) = 54$ $\quad n(A \cap B) = 22$ $\quad n(A \cup B) = 85$

$n(A \cap B \cap C) = 4$

$n(A \cap C) = 15$

$n(B \cap C) = 16$

$n(C) = 44$

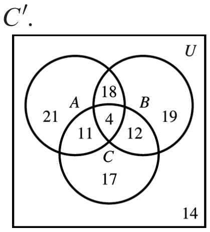
$n(B') = 63$

Start with $A \cap B \cap C$. We have $n(A \cap C) = 15$, of which 4 elements are in $A \cap B \cap C$, so $n(A \cap B' \cap C) = 11$. $n(B \cap C) = 16$, of which 4 elements are in $A \cap B \cap C$, so $n(B \cap C \cap A') = 12$. $n(C) = 44$, so 17 elements are in $C \cap A' \cap B'$. $n(A \cap B) = 22$, so 18 elements are in $A \cap B \cap C'$. $n(A) = 54$, so $54 - 11 - 18 - 4 = 21$ elements are in $A \cap B' \cap C'$.

Now use the union rule.

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 85 &= 54 + n(B) - 22 \\ 53 &= n(B) \end{aligned}$$

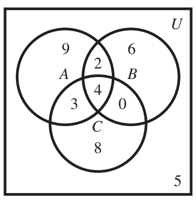
This leaves 19 elements in $B \cap A' \cap C'$. $n(B') = 63$, of which $21 + 11 + 17 = 49$ are accounted for, leaving 14 elements in $A' \cap B' \cap C'$.



31. $n(A \cap B) = 6$ $\quad n(A \cap B \cap C) = 4$
$n(A \cap C) = 7$ $\quad n(B \cap C) = 4$
$n(A \cap C') = 11$ $\quad n(B \cap C') = 8$
$n(C) = 15$ $\quad n(A' \cap B' \cap C') = 5$

Start with $n(A \cap B) = 6$ and $n(A \cap B \cap C) = 4$ to get $6 - 4 = 2$ in that portion of $A \cap B$ outside of C. From $n(B \cap C) = 4$, there are $4 - 4 = 0$ elements in that portion of $B \cap C$ outside of A. Use $n(A \cap C) = 7$ to get $7 - 4 = 3$ elements in that portion of $A \cap C$ outside of B.

Since $n(A \cap C') = 11$, there are $11 - 2 = 9$ elements in that part of A outside of B and C. Use $n(B \cap C') = 8$ to get $8 - 2 = 6$ elements in that part of B outside of A and C. Since $n(C) = 15$, there are $15 - 3 - 4 - 0 = 8$ elements in C outside of A and B. Finally, 5 must be outside all three sets, since $n(A' \cap B' \cap C') = 5$.



32. $n(A) = 13$ $\quad n(A \cap B \cap C) = 4$
$n(A \cap C) = 6$ $\quad n(A \cap B') = 6$
$n(B \cap C) = 6$ $\quad n(B \cap C') = 11$
$n(B \cup C) = 22$ $\quad n(A' \cap B' \cap C') = 5$

Start with the regions $A \cap B \cap C$ and $A' \cap B' \cap C'$.

$n(B \cap C) = 6$, leaving 2 elements in $B \cap C \cap A'$.

$n(A \cap C) = 6$, leaving 2 elements in $A \cap C \cap B'$.

$n(A \cap B') = 6$, leaving 4 elements in $A \cap B' \cap C'$.

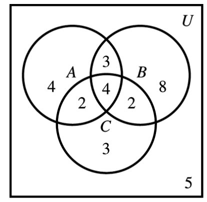
$n(A) = 13$, leaving 3 elements in $A \cap B \cap C'$.

$n(B \cap C') = 11$, leaving 8 elements in $B \cap C' \cap A'$.

Now use the union rule.

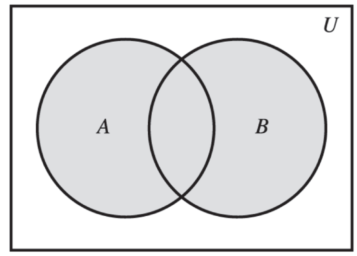
$$\begin{aligned} n(B \cup C) &= n(B) + n(C) - n(B \cap C) \\ 22 &= 17 + n(C) - 6 \\ 11 &= n(C) \end{aligned}$$

This leaves 3 elements in $C \cap A' \cap B'$.



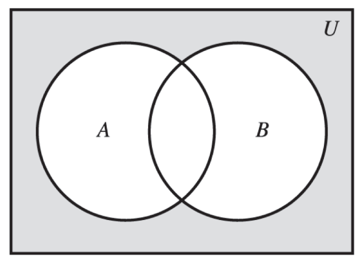
33. $(A \cup B)' = A' \cap B'$

For $(A \cup B)'$, first find $A \cup B$.



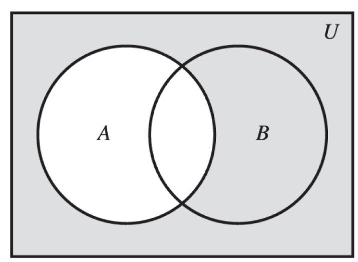
$A \cup B$

Now find $(A \cup B)'$, the region outside $A \cup B$.

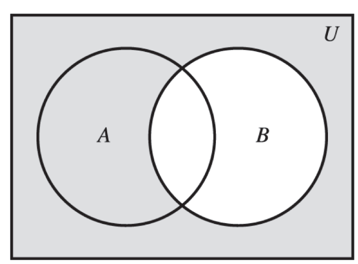


$(A \cup B)'$

For $A' \cap B'$, first find A' and B' individually.



A'

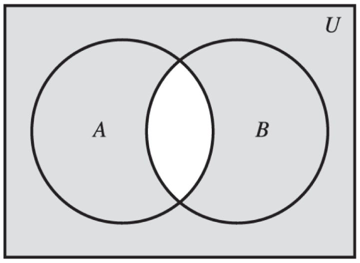


B'

Then $A' \cap B'$ is the region where A' and B' overlap, which is the entire region outside $A \cup B$ (the same result as in the second diagram). Therefore,

$$(A \cup B)' = A' \cap B'.$$

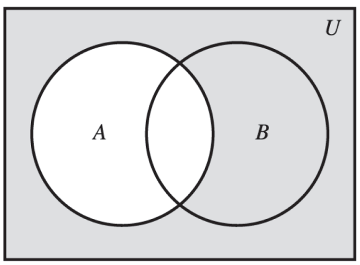
34. $(A \cap B)'$ is the complement of the intersection of A and B; hence it contains all elements not in $A \cap B$.



$(A \cap B)'$

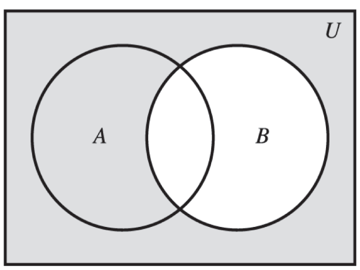
$A' \cup B'$ is the union of the complements of A and B; hence it contains any element that is either not in A or not in B.

A' is the set of all elements in U that are not in A.



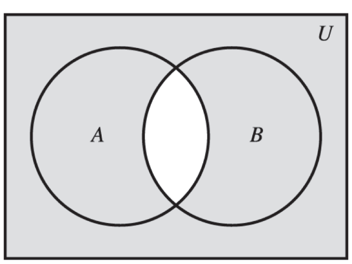
A'

B' is the set of all elements in U that are not in B.



B'

Form the union of A' and B'.



$A' \cup B'$

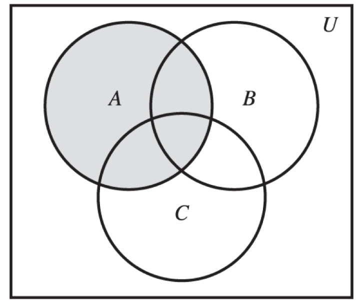
The Venn diagrams show that

$$(A \cap B)' = A' \cup B',$$

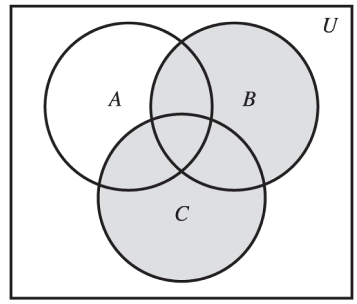
as claimed.

35. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

First find A and $B \cup C$ individually.

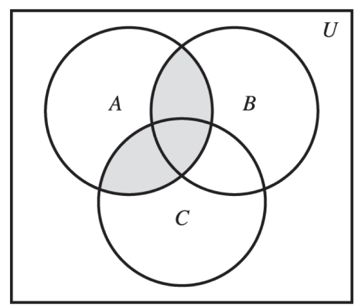


A



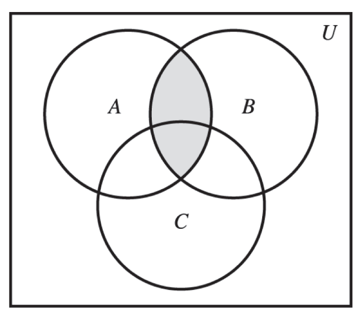
$B \cup C$

Then $A \cap (B \cup C)$ is the region where the above two diagram overlap.

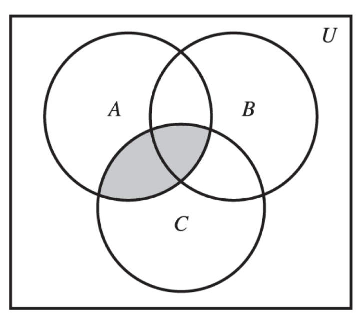


$A \cap (B \cup C)$

Next find $A \cap B$ and $A \cap C$ individually.

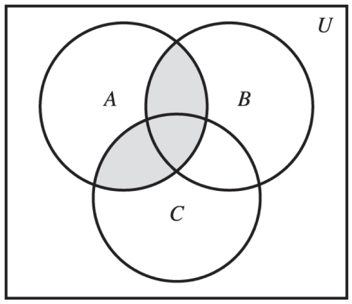


$A \cap B$



$A \cap C$

Then $(A \cap B) \cup (A \cap C)$ is the union of the above two diagrams.

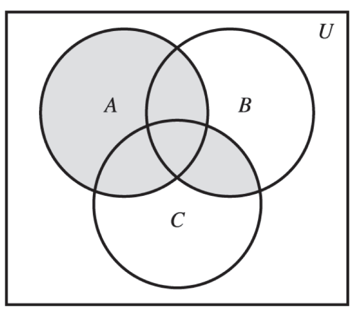


$(A \cap B) \cup (A \cap C)$

The Venn diagram for $A \cap (B \cup C)$ is identical to the Venn diagram for $(A \cap B) \cup (A \cap C)$, so conclude that

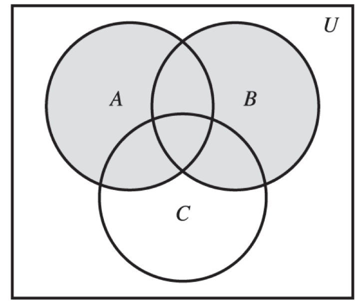
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

36. $A \cup (B \cap C)$ contains all the elements in A or in both B and C.

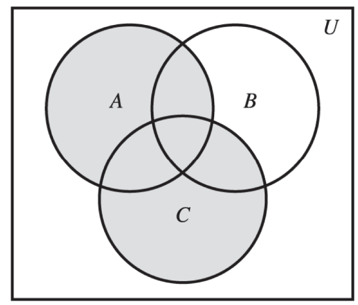


$A \cup (B \cap C)$

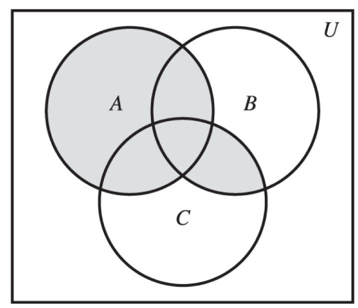
$(A \cup B) \cap (A \cup C)$ contains the intersection $A \cup B$ and $A \cup C$.



$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$

Comparing the Venn diagrams, we see that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$

as claimed.

37. Prove

$$\begin{aligned} &n(A \cup B \cup C) \\ &\quad = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) \\ &\qquad\qquad - n(B \cap C) + n(A \cap B \cap C) \end{aligned}$$

$$\begin{aligned} &n(A \cup B \cup C) \\ &\quad = n[A \cup (B \cup C)] \\ &\quad = n(A) + n(B \cup C) - n[A \cap (B \cup C)] \\ &\quad = n(A) + n(B) + n(C) - n(B \cap C) \\ &\qquad\qquad - n[(A \cap B) \cup (A \cap C)] \\ &\quad = n(A) + n(B) + n(C) - n(B \cap C) \\ &\qquad\qquad - \{n(A \cap B) + n(A \cap C) \\ &\qquad\qquad - n[(A \cap B) \cap (A \cap C)]\} \\ &\quad = n(A) + n(B) + n(C) - n(B \cap C) - n(A \cap B) \\ &\qquad\qquad - n(A \cap C) + n(A \cap B \cap C) \end{aligned}$$

38. Let M be the set of those who use a microwave oven, E be the set of those who use an electric range, and G be the set of those who use a gas range. We are given the following information.

$n(U) = 140$
$n(M) = 58$
$n(E) = 63$
$n(G) = 58$
$n(M \cap E) = 19$
$n(M \cap G) = 17$
$n(G \cap E) = 4$
$n(M \cap G \cap E) = 1$
$n(M' \cap G' \cap E') = 2$

Since $n(M \cap G \cap E) = 1,$ there is 1 element in the region where the three sets overlap.

Since $n(M \cap E) = 19,$ there are $19 - 1 = 18$ elements in $M \cap E$ but not in $M \cap G \cap E.$

Since $n(M \cap G) = 17,$ there are $17 - 1 = 16$ elements in $M \cap G$ but not in $M \cap G \cap E.$

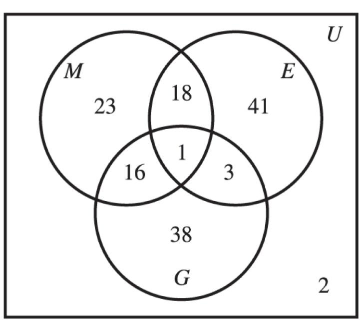
Since $n(G \cap E) = 4,$ there are $4 - 1 = 3$ elements in $G \cap E$ but not in $M \cap G \cap E.$

Now consider $n(M) = 58.$ So far we have $16 + 1 + 18 = 35$ in M; there must be another $58 - 35 = 23$ in M not yet counted.

Similarly, $n(E) = 63;$ we have $18 + 1 + 3 = 22$ counted so far. There must be $63 - 22 = 41$ more in E not yet counted.

Also, $n(G) = 58;$ we have $16 + 1 + 3 = 20$ counted so far. There must be $58 - 20 = 38$ more in G not yet counted.

Lastly, $n(M' \cap G' \cap E') = 2$ indicates that there are 2 elements outside of all three sets.



Note that the numbers in the Venn diagram add up to 142 even though $n(U) = 140.$ Jeff has made some error, and he should definitely be reassigned.

39. Let A be the set of trucks that carried early peaches, B be the set of trucks that carried late peaches, and C be the set of trucks that carried extra late peaches. We are given the following information.

$n(A) = 34 \qquad n(B) = 61 \qquad n(C) = 50$
$n(A \cap B) = 25$
$n(B \cap C) = 30$
$n(A \cap C) = 8$
$n(A \cap B \cap C) = 6$
$n(A' \cap B' \cap C') = 9$

Start with $A \cap B \cap C.$

We know that $n(A \cap B \cap C) = 6.$

Since $n(A \cap B) = 25,$ the number in $A \cap B$ but not in C is $25 - 6 = 19.$

Since $n(B \cap C) = 30,$ the number in $B \cap C$ but not in A is $30 - 6 = 24.$

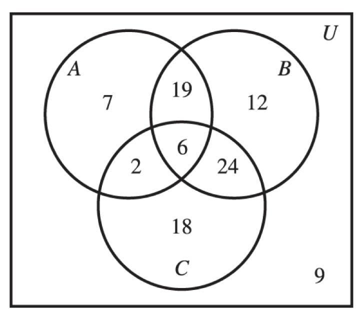
Since $n(A \cap C) = 8,$ the number in $A \cap C$ but not in B is $8 - 6 = 2.$

Since $n(A) = 34,$ the number in A but not in B or C is $34 - (19 + 6 + 2) = 7.$

Since $n(B) = 61,$ the number in B but not in A or C is $61 - (19 + 6 + 24) = 12.$

Since $n(C) = 50,$ the number in C but not in A or B is $50 - (24 + 6 + 2) = 18.$

Since $n(A' \cap B' \cap C') = 9,$ the number outside $A \cup B \cup C$ is 9.



(a) From the Venn diagram, 12 trucks carried only late peaches.

(b) From the Venn diagram, 18 trucks carried only extra late peaches.

(c) From the Venn diagram, $7 + 12 + 18 = 37$ trucks carried only one type of peach.

(d) From the Venn diagram, $6 + 2 + 19 + 24 + 7 + 12 + 18 + 9 = 97$ trucks went out during the week.

40. **(a)** $n(Y \cap R) = 40$ since 40 is the number in the table where the Y row and the R column meet.

(b) $n(M \cap D) = 30$ since 30 is the number in the table where the M row and the D column meet.

(c) $n(D \cap Y) = 15$ and $n(M) = 80$ since that is the total in the M row. $n(M \cap (D \cap Y)) = 0$ since no person can simultaneously have an age in the range 21–25 *and* have an age in the range 26–35. By the union rule for sets,

$$\begin{aligned} n(M \cup (D \cap Y)) &= n(M) + n(D \cap Y) - n(M \cap (D \cap Y)) \\ &= 80 + 15 - 0 \\ &= 95. \end{aligned}$$

(d) $Y' \cap (D \cup N)$ consists of all people in the D column or in the N column who are at the same time not in the Y row. Therefore,

$$\begin{aligned} n(Y' \cap (D \cup N)) &= 30 + 50 + 20 + 10 \\ &= 110. \end{aligned}$$

(e) $n(N) = 45$
$n(O) = 70$
$n(O') = 220 - 70 = 150$
$n(O' \cap N) = 15 + 20 = 35$
By the union rule,

$$\begin{aligned} n(O' \cup N) &= n(O') + n(N) - n(O' \cap N) \\ &= 150 + 45 - 35 \\ &= 160. \end{aligned}$$

(f) $M' \cap (R' \cap N')$ consists of all people who are not in the R column and not in the N column and who are at the same time not in the M row. Therefore,

$$n(M' \cap (R' \cap N')) = 15 + 50 = 65.$$

(g) $M \cup (D \cap Y)$ consists of all people age 21–25 who drink diet cola *or* anyone age 26–35.

41. **(a)** $n(Y \cap B) = 2$ since 2 is the number in the table where the Y row and the B column meet.

(b) $$\begin{aligned} n(M \cup A) &= n(M) + n(A) - n(M \cap A) \\ &= 33 + 41 - 14 = 60 \end{aligned}$$

(c) $n[Y \cap (S \cup B)] = 6 + 2 = 8$

(d)

$$\begin{aligned} &n[O' \cup (S \cup A)] \\ &= n(O') + n(S \cup A) - n[O' \cap (S \cup A)] \\ &= (23 + 33) + (52 + 41) - (6 + 14 + 15 + 14) \\ &= 100 \end{aligned}$$

(e) Since is $M' \cup O'$ is the entire set, $(M' \cup O') \cap B = B.$ Therefore,

$$n[(M' \cup O') \cap B] = n(B) = 27.$$

(f) $Y \cap (S \cup B)$ is the set of all bank customers who are of age 18–29 and who invest in stocks or bonds.

42. We start with the innermost region, which is the number of professors who invested in stocks and bonds and certificates of deposit (CDs). Since this is our unknown, place an x in this region. If 80 invested in stocks and bonds, then $80 - x$ invested in only stocks and bonds.

If 83 invested in bonds and CDs, then $83 - x$ invested in only bonds and CDs. If 85 invested in stocks and CDs, then $85 - x$ invested in only stocks and CDs. If 111 invested in stocks, then the number who invested in only stocks is:

$$111 - [(80 - x) + (85 - x) + x] = x - 54.$$

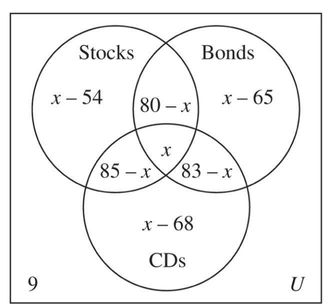
If 98 invested in bonds, then the number who invested in only bonds is;

$$98 - [(80 - x) + (83 - x) + x] = x - 65.$$

If 100 invested in CDs, then the number who invested in only CDs is:

$$100 - [(85 - x) + (83 - x) + x] = x - 68.$$

There are 9 who did not invest in any of the three, so place a 9 outside all three circles. Now, the sum of all the regions is 150, so



$$\begin{aligned} 150 &= (x - 54) + (80 - x) + (x - 65) + x \\ &\quad + (85 - x) + (83 - x) + (x - 68) + 9 \\ 150 &= 70 + x \\ x &= 80 \end{aligned}$$

80 professors invested in stocks and bonds and certificates of deposits.

43. Let T be the set of all tall pea plants, G be the set of plants with green peas, and S be the set of plants with smooth peas. We are given the following information.

$n(U) = 50 \quad n(T) = 22 \quad n(G) = 25 \quad n(S) = 39$

$n(T \cap G) = 9$

$n(G \cap S) = 20$

$n(T \cap G \cap S) = 6$

$n(T' \cap G' \cap S') = 4$

Start by filling in the Venn Diagram with the numbers for the last two regions, $T \cap G \cap S$ and $T' \cap G' \cap S'$, as shown below.

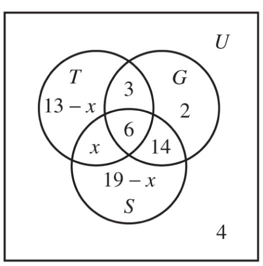
With $n(T \cap G) = 9$, this leaves $n(T \cap G \cap S') = 9 - 6 = 3$.

With $n(G \cap S) = 20$, this leaves $n(T' \cap G \cap S) = 20 - 6 = 14$.

Since $n(G) = 25$, $n\ (T' \cap G \cap S') = 25 - 3 - 6 - 14 = 2$.

With no other regions that we can calculate, denote by x the number in $T \cap G' \cap S'$. Then $n(T \cap G' \cap S') = 22 - 3 - 6 - x = 13 - x$, and $n(T' \cap G' \cap S) = 39 - 6 - 14 - x = 19 - x$, as shown. Summing the values for all eight regions,

$$\begin{aligned}(13 - x) + 3 + 2 + x + 6 + 14 + (19 - x) + 4 &= 50\\ 61 - x &= 50\\ x &= 11\end{aligned}$$



(a) $n(T \cap S) = 11 + 6 = 17$

(b) $n(T \cap G' \cap S') = 13 - x = 13 - 11 = 2$

(c) $n(T' \cap G \cap S) = 14$

44. **(a)** The blood has the A antigen but is Rh negative and has no B antigen. This blood type is A-negative.

(b) Both A and B antigens are present and the blood is Rh negative. This blood type is AB-negative.

(c) Only the B antigen is present. This blood type is B-negative.

(d) Both A and Rh antigens are present. This is blood type is A-positive.

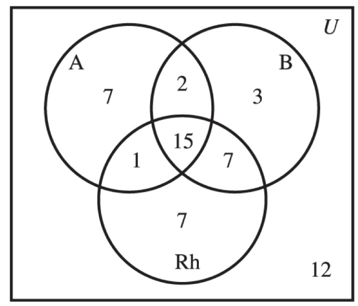
(e) All antigens are present. This blood type is AB-positive.

(f) Both B and Rh antigens are present. This blood type is B-positive.

(g) Only the Rh antigen is present. This blood type is O-positive.

(h) No antigens at all are present. This blood type is O-negative.

45. First fill in the Venn diagram, starting with the region common to all three sets.



(a) The total of these numbers in the diagram is 54.

(b) $7 + 3 + 7 = 17$ had only one antigen.

(c) $1 + 2 + 7 = 10$ had exactly two antigens.

(d) A person with O-positive blood has only the Rh antigen, so this number is 7.

(e) A person with AB-positive blood has all three antigens, so this number is 15.

(f) A person with B-negative blood has only the B antigen, so this number is 3.

(g) A person with O-negative blood has none of the antigens. There are 12 such people.

(h) A person with A-positive blood has the A and Rh antigens, but not the B-antigen. The number is 1.

46. Extend the table to include totals for each row and column.

	W	*B*	*I*	*A*	**Totals**
F	1,055,221	141,369	6407	21,325	1,224,322
M	1,022,328	148,602	7630	23,382	1,201,942
Totals	2,077,549	289,971	14,037	44,707	2,426,264

(a) $n(F)$ is the total for the first row in the table. $n(F) = 1,224,322$. Thus, there are 1,224,322 people in the set F.

(b) $n(F \cap (I \cup A))$ is the sum of the entries in the first row in the I and A columns. Therefore, $n(F \cap (I \cup A)) = n(F \cap I) + n(F \cap A) = 27{,}732.$ Therefore, there are 27,732 people in the set $F \cap (I \cup A)$.

(c) $n(M \cup B) = n(M) + n(B) - n(M \cap B)$
$= 1{,}201{,}942 + 289{,}971 - 148{,}602$
$= 1{,}343{,}311.$

There are $1{,}343{,}311$ people in the set $M \cup B$.

(d) $W' \cup I' \cup A'$ is the universe, since each person is either *not* white, or *not* American Indian, or *not* Asian or Pacific Islander. Thus, there are $2{,}426{,}264$ people in the set $W' \cup I' \cup A'$.

(e) Females who are either American Indian or Pacific Islander.

47. Extend the table to include totals for each row and each column.

	H	F	Total
A	95	34	129
B	41	38	79
C	9	7	16
D	202	150	352
Total	347	229	576

(a) $n(A \cap F)$ is the entry in the table that is in both row A and column F. Thus, there are 34 players in the set $A \cap F$.

(b) Since all players in the set C are either in set H or set $F, C \cap (H \cup F) = C$. Thus, $n(C \cap (H \cup F)) = n(C) = 16,$ the total for row C. There are 16 players in the set $C \cap (H \cup F)$.

(c) $n(D \cup F) = n(D) + n(F) - n(D \cap F)$
$= 352 + 229 - 150$
$= 431$

(d) $B' \cap C'$ is the set of players who are both *not* in B and *not* in C. Thus, $B' \cap C' = A \cup D$, and since A and D are disjoint, $n(A \cup D) = n(A) + n(D) = 129 + 352 = 481.$

There are 481 players in the set $B' \cap C'$.

48. Use the following table:

	A	B	C	D	Totals
O	14,322	12,097	7884	1202	35,505
E	59,401	51,965	42,225	11,749	165,340
M	688	922	920	0	2530
Totals	74,411	64,984	51,029	12,951	203,375

(a) $n(A \cup B)$ is the total number of female personnel serving in the Army or the Air Force: $n(A \cup B) = n(A) + n(B) = 139{,}395.$

(b) $E \cup (C \cup D)$ includes all enlisted personnel, and all personnel in the Navy and Marines. In order not to count any person twice, we take $n(E)$ and add $n(C \cap E')$ and $n(D \cap E')$. This gives us $165{,}340 + (7884 + 920) + (1202 + 0) = 175{,}346.$ There are 175,346 people in the set $E \cup (C \cup D)$.

(c) $O' \cap M' = E,$ since the enlisted are the only group who are both not officers and not cadets or midshipmen. Thus $n(O' \cap M') = n(E) = 165{,}340.$ There are 165,340 people in the set $O' \cap M'$.

49. Reading directly from the table, $n(A \cap B) = 110.6.$ Thus, there are 110.6 million people in the set $A \cap B$.

50. $n(G \cup B) = n(G) + n(B) - n(G \cap B)$
$= 80.4 + 52.6 - 10.3$
$= 122.7$

Thus, there are 122.7 million people in the set $G \cup B$.

51. $n(G \cup (C \cap H)) = n(G) + n(C \cap H)$
$= 80.4 + 5.0$
$= 85.4$

There are 85.4 million people in the set $G \cup (C \cap H)$.

52. The only intersection of the set F and the set $B \cup H$ is the set $F \cap B$. Therefore, $n(F \cap (B \cup H)) = n(F \cap B) = 37.6.$ There are 37.6 million people in the set $F \cap (B \cup H)$.

53. $n(H \cup D) = n(H) + n(D) - n(H \cap D)$
$= 53.6 + 19.6 - 2.2$
$= 71.0$

There are 71.0 million people in the set $H \cup D$.

54. First of all, $A' \cap C' = B \cup D \cup E$ since the only people *not* in set A and *not* in the set C are the people in set B or set D or set E. Second $G' = F \cup H$, since the only people *not* in the set G are either in the set F or the set H. Thus, the set $G' \cap (A' \cap C')$ consists of people in either F or H and also in B, D, or E. Therefore,

$$\begin{aligned} n(G' \cap (A' \cap C')) &= n(F \cap B) + n(F \cap D) \\ &\quad + n(F \cap E) + n(H \cap B) \\ &\quad + n(H \cap D) + n(H \cap E) \\ &= 37.6 + 13.1 + 2.2 + 4.7 \\ &\quad + 2.2 + 0.3 \\ &= 60.1 \end{aligned}$$

There are 60.1 million people in the set $G' \cap (A' \cap C')$.

For Exercises 55 through 58, use the following table, where the numbers are in thousands and the table has been extended to include row and column totals.

	W	*B*	*H*	*A*	**Totals**
N	54,205	13,547	12,021	3518	83,291
M	106,517	9577	16,111	6741	138,946
I	11,968	1740	1068	507	15,283
D	23,046	4590	3477	665	31,778
Totals	195,736	29,454	32,677	11,431	269,298

55. $N \cap (B \cup H)$ is the set of Blacks or Hispanics who never married. These people are located in the first row of the table, in the B and H columns, so $n(N \cap (B \cup H)) = 13{,}547 + 12{,}021 = 25{,}568$; since the table values are in thousands, this set contains 25,568,000 people.

56. $(M \cup I) \cap A$ is the set of married or widowed people who are also Asian/Pacific Islanders. They are found in the M and I rows of the table in the A column, so $n(M \cup I) \cap A) = 6741 + 507 = 7{,}248$; since the table values are in thousands, this set contains 7,248,000 people.

57. $(D \cup W) \cap A'$ is the set of Whites or Divorced/separated people who are not Asian/Pacific Islanders. The number of these people is found by adding the total of the W column to the total of the D row and then subtracting the number of Divorced/separated Whites (so we don't count them twice) and subtracting the number of Divorced/separated Asians/Pacific Islanders (whom we don't want to count).

$$\begin{aligned} n((D \cup W) \cap A') &= 195{,}736 + 31{,}778 - 23{,}046 - 665 \\ &= 203{,}803; \end{aligned}$$

since the table values are in thousands, this set contains 203,803,000 people.

58. $M' \cap (B \cup A)$ is the set of unmarried people who are either Blacks or Asian/Pacific Islanders. Their number can be found by adding the totals of the B and A columns and subtracting the number of married Blacks and the number of married Asian/Pacific Islanders.

$$\begin{aligned} n(M' \cap (B \cup A)) &= 29{,}454 + 11{,}431 - 9577 - 6741 \\ &= 24{,}567; \end{aligned}$$

since the table values are in thousands, this set contains 24,567,000 people.

59. Let W be the set of women, C be the set of those who speak Cantonese, and F be the set of those who set off firecrackers. We are given the following information.

$n(W) = 120 \quad n(C) = 150 \quad n(F) = 170$
$n(W' \cap C) = 108 \quad n(W' \cap F') = 100$
$n(W \cap C' \cap F) = 18$
$n(W' \cap C' \cap F') = 78$
$n(W \cap C \cap F) = 30$

Note that

$$\begin{aligned} n(W' \cap C \cap F') &= n(W' \cap F') - n(W' \cap C' \cap F') \\ &= 100 - 78 \\ &= 22. \end{aligned}$$

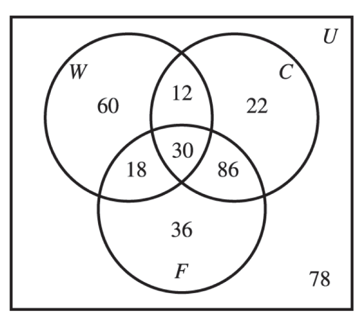
Furthermore,

$$\begin{aligned} n(W' \cap C \cap F) &= n(W' \cap C) - n(W' \cap C \cap F') \\ &= 108 - 22 \\ &= 86. \end{aligned}$$

We now have

$$\begin{aligned} &n(W \cap C \cap F') \\ &= n(C) - n(W' \cap C \cap F) - n(W \cap C \cap F) \\ &\quad - n(W' \cap C \cap F') \\ &= 150 - 86 - 30 - 22 = 12. \end{aligned}$$

With all of the overlaps of W, C, and F determined, we can now compute $n(W \cap C' \cap F') = 60$ and $n(W' \cap C' \cap F) = 36$.



(a) Adding up the disjoint components, we find the total attendance to be

$$60 + 12 + 18 + 30 + 22 + 86 + 36 + 78 = 342.$$

(b) $n(C') = 342 - n(C) = 342 - 150 = 192$

(c) $n(W \cap F') = 60 + 12 = 72$

(d) $n(W' \cap C \cap F) = 86$

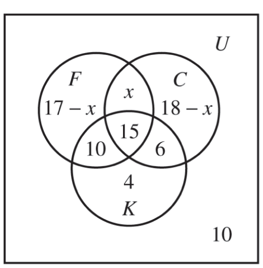
60. Let F be the set of people who brought food, C be the set of those who brought costumes, and K be the set of those who brought crafts. We are given the following information.

$$n(U) = 75$$
$$n(F \cap C \cap K) = 15$$
$$n(F \cap K) = 25$$
$$n(F) = 42$$
$$n(K) = 35$$
$$n(K \cap C') = 14$$
$$n(F' \cap C' \cap K') = 10$$
$$n(C \cap K') = 18$$

Fill in regions in the Venn diagram in this order:

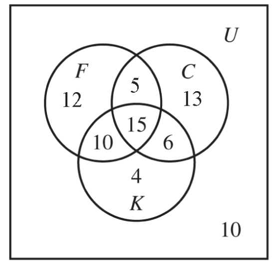
Put 15 in the region $F \cap C \cap K$. Since $n(F \cap K) = 25$, we can put $25 - 15 = 10$ in $F \cap C' \cap K$. Since $n(K \cap C') = 14$, we can now put $14 - 10 = 4$ in $K \cap F' \cap C'$. Since $n(K) = 35$, we can then put $35 - 10 - 15 - 14 = 6$ in $C \cap K \cap F'$. Now we have run out of numbers that we can compute directly, so we put the unknown x into $F \cap C \cap K'$.

Since $n(F) = 42$ we can now put $42 - 10 - 15 - x$ or $17 - x$ into $F \cap C' \cap K'$. Since $n(C \cap K') = 18$, we can put $18 - x$ in $C \cap F' \cap K'$. Finally, since 10 families brought none of the three items, we can put a 10 outside the circles. The Venn diagram now looks like this:



To find x, we note that since there were 75 families and 10 brought none of the three items, the total of the seven inner regions must be $75 - 10 = 65$.

$17 - x + 18 - x + 4 + 10 + x + 6 + 15 = 65$ implies that $70 - x = 65$ or $x = 5$. This allows us to fill in all the missing regions.



We can now read the answers to the questions from the completed diagram.

(a) $n(C \cap F) = 15 + 5 = 20$

(b) $n(C) = 5 + 13 + 15 + 6 = 39$

(c) $n(F \cap C') = 12 + 10 = 22$

(d) $n(K') = 12 + 5 + 13 + 10 = 40$

(e) $n(F \cup C) = 12 + 5 + 13 + 10 + 15 + 6$
$= 61$

61. Let F be the set of fat chickens (so F' is the set of thin chickens), R be the set of red chickens (so R' is the set of brown chickens), and M be the set of male chickens, or roosters (so M' is the set of female chickens, or hens). We are given the following information.

$$n(F \cap R \cap M) = 9$$
$$n(F' \cap R' \cap M') = 13$$
$$n(R \cap M) = 15$$
$$n(F' \cap R) = 11$$
$$n(R \cap M') = 17$$
$$n(F) = 56$$
$$n(M) = 41$$
$$n(M') = 48$$

First, note that $n(M) + n(M') = n(U) = 89$, the total number of chickens.

Since $n(R \cap M) = 15$, $n(F' \cap R \cap M)$
$= 15 - 9 = 6.$

Since $n(F' \cap R) = 11$, $n(F' \cap R \cap M')$
$= 11 - 6 = 5.$

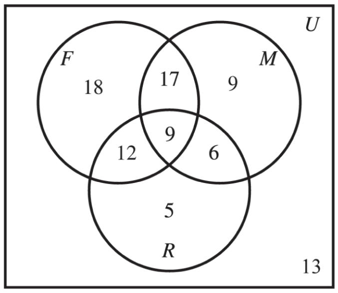
Since $n(R \cap M') = 17$, $n(F \cap R \cap M')$
$= 17 - 5 = 12.$

Since $n(M') = 48$, $n(F \cap R' \cap M')$
$= 48 - (12 + 5 + 13) = 18.$

Since $n(F) = 56$, $n(F \cap R' \cap M)$
$= 56 - (18 + 12 + 9) = 17.$

And, finally, since $n(M) = 41$,

$$\begin{aligned} n(F' \cap R' \cap M) &= 41 - (17 + 9 + 6) \\ &= 9. \end{aligned}$$



(a) $\begin{aligned} n(U) &= 18 + 9 + 5 + 17 + 6 \\ &\quad + 12 + 9 + 13 \\ &= 89 \end{aligned}$

(b) $\begin{aligned} n(R) &= n(R \cap M) + n(R \cap M') \\ &= 15 + 17 \\ &= 32 \end{aligned}$

(c) $\begin{aligned} n(F \cap M) &= n(F \cap R \cap M) \\ &\quad + n(F \cap R' \cap M) \\ &= 17 + 9 \\ &= 26 \end{aligned}$

(d) $\begin{aligned} n(F \cap M') &= n(F) - n(F \cap M) \\ &= 56 - 26 \\ &= 30 \end{aligned}$

(e) $\begin{aligned} n(F' \cap R') &= n(F' \cap R' \cap M) \\ &\quad + n(F' \cap R' \cap M') \\ &= 9 + 13 \\ &= 22 \end{aligned}$

(f) $\begin{aligned} &n(F \cap R) \\ &= n(F \cap R \cap M) + n(F \cap R \cap M') \\ &= 9 + 12 = 21 \end{aligned}$

7.3 Introduction to Probability

Your Turn 1

The sample space of equally likely outcomes is $\{hh, ht, th, tt\}$.

Your Turn 2

The sample space for tossing two coins is $\{hh, ht, th, tt\}$. The event E: "the coins show exactly one head" is $E = \{ht, th\}$.

Your Turn 3

E: worker is under 20, so E': worker is 20 or over.

F: worker is white, so F': worker is not white.

Thus, $E' \cap F'$ is the event that the worker is 20 or over and is not white.

Your Turn 4

The sample space S is $S = \{1, 2, 3, 4, 5, 6\}$. The event H that the die shows a number less than 5 is $H = \{1, 2, 3, 4\}$. The probability of H is

$$P(H) = \frac{n(H)}{n(S)} = \frac{4}{6} = \frac{2}{3}.$$

Your Turn 5

In a standard 52-card deck there are 4 jacks and 4 kings, so $P(\text{jack or king}) = \frac{8}{52} = \frac{2}{13}$.

7.3 Exercises

3. The sample space is the set of the twelve months, {January, February, March, . . ., December}.

4. List the days in April.

$$S = \{1, 2, 3, 4, \ldots, 29, 30\}$$

5. The possible number of points earned could be any whole number from 0 to 80. The sample space is the set

$$\{0, 1, 2, 3, \ldots, 79, 80\}.$$

6. List the possible number of hours of TV watching in a day.

$$S = \{0, 1, 2, 3, \ldots, 23, 24\}$$

7. The possible decisions are to go ahead with a new oil shale plant or to cancel it. The sample space is the set {go ahead, cancel}.

8. Let u = up and d = down. There are $2^3 = 8$ possible 3-day outcomes, so

$$S = \{uuu, uud, udu, duu, ddu, dud, udd, ddd\}.$$

9. Let h = heads and t = tails for the coin; the die can display 6 different numbers. There are 12 possible outcomes in the sample space, which is the set

$$\{(h,1), (h,2), (h,3), (h,4), (h,5), (h,6), \\ (t,1), (t,2), (t,3), (t,4), (t,5), (t,6)\}.$$

10. There are $5 \cdot 5 = 25$ possible outcomes.

$$\begin{aligned} S = \{&(1,1), (1,2), (1,3), (1,4), (1,5), \\ &(2,1), (2,2), (2,3), (2,4), (2,5), \\ &(3,1), (3,2), (3,3), (3,4), (3,5), \\ &(4,1), (4,2), (4,3), (4,4), (4,5), \\ &(5,1), (5,2), (5,3), (5,4), (5,5)\} \end{aligned}$$

13. Use the first letter of each name. The sample space is the set

$$S = \{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE\}.$$

$n(S) = 10.$ Assuming the committee is selected at random, the outcomes are equally likely.

(a) One of the committee members must be Chinn. This event is {AC, BC, CD, CE}.

(b) Alam, Bartolini, and Chinn may be on any committee; Dickson and Ellsberg may not be on the same committee. This event is

{AB, AC, AD, AE, BC, BD, BE, CD, CE}.

(c) Both Alam and Chinn are on the committee. This event is {AC}.

14. $S =$ {(CA, CO, NJ), (CA, CO, NY), (CA, CO, UT) (CA, NJ, NY), (CA, NJ, UT), (CA, NY, UT), (CO, NJ, NY), (CO, NJ, UT), (CO, NY, UT), (NJ, NY, UT)}

$N(s) = 10$

Assuming the states are chosen at random, the outcomes are equally likely.

(a) Of the states listed, the ones that border an ocean are CA, NJ, and NY. Therefore, the event "all three states border an ocean" is the set {(CA, NJ, NY)}.

(b) The states that border an ocean are CA, NJ, and NY. Therefore, the event "exactly two of the three states border an ocean" is the set {(CA, CO, NJ), (CA, CO, NY), (CA, NJ, UT), (CA, NY, UT), (CO, NJ, NY), (NJ, NY, UT)}.

(c) The states that lie west of the Mississippi River are CA, CO, and UT. Therefore, the event "exactly one of the three states is west of the Mississippi River" is the set

{(CA, NJ, NY), (CO, NJ, NY), (NJ, NY, UT)}.

15. Each outcome consists of two of the numbers 1, 2, 3, 4, and 5, without regard for order. For example, let (2, 5) represent the outcome that the slips of paper marked with 2 and 5 are drawn. There are ten equally likely outcomes in this sample space, which is

$$S = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}.$$

(a) Both numbers in the outcome pair are even. This event is {(2, 4)}, which is called a simple event since it consists of only one outcome.

(b) One number in the pair is even and the other number is odd. This event is

$$\{(1,2), (1,4), (2,3), (2,5), (3,4), (4,5)\}.$$

(c) Each slip of paper has a different number written on it, so it is not possible to draw two slips marked with the same number. This event is $\emptyset$, which is called an impossible event since it contains no outcomes.

16. Let $w =$ wrong; $c =$ correct.

$$S = \{www, wwc, wcw, cww, ccw, cwc, wcc, ccc\}$$

$n(S) = 8.$ The problem states the outcomes are equally likely.

(a) The student gets three answers wrong. This event is written $\{www\}$.

(b) The student gets exactly two answers correct. Since either the first, second, or third answer can be wrong, this can happen in three ways. The event is written $\{ccw, cwc, wcc\}$.

(c) The student gets only the first answer correct. The second and third answers must be wrong. This event is written $\{cww\}$.

17. $$S = \{HH, THH, HTH, TTHH, THTH, HTTH, TTTH, TTHT, THTT, HTTT, TTTT\}$$

$n(S) = 11.$ The outcomes are not equally likely.

(a) The coin is tossed four times. This event is written $\{TTHH, THTH, HTTH, TTTH, TTHT, THTT, HTTT, TTTT\}$.

(b) Exactly two heads are tossed. This event is written $\{HH, THH, HTH, TTHH, THTH, HTTH\}$.

(c) No heads are tossed. This event is written $\{TTTT\}$.

18. There are 4 possibilities for the first choice and 5 for the second choice. The sample space is

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,5), (3,1), (3,2), (3,3), (3,4), (3,5), (4,1), (4,2), (4,3), (4,4), (4,5)\}.$$

$n(S) = 20.$ Yes, the outcomes are equally likely.

(a) The first choice must be 2 or 4; the second can range from 1 to 5:

$$\{(2,1), (2,2), (2,3), (2,4), (2,5), (4,1), (4,2), (4,3), (4,4), (4,5)\}.$$

(b) The first choice can range from 1 to 4; the second must be 2 or 4:

$$\{(1,2), (1,4), (2,2), (2,4), (3,2), (3,4), (4,2), (4,4)\}.$$

(c) The choices must add up to 5:

$$\{(1,4), (2,3), (3,2), (4,1)\}.$$

(d) It is not possible for the sum to be 1: $\emptyset$.

For Exercises 19–24, use the sample space

$$S = \{1, 2, 3, 4, 5, 6\}.$$

19. "Getting a 2" is the event $E = \{2\}$, so $n(E) = 1$ and $n(S) = 6$.

If all the outcomes in a sample space S are equally likely, then the probability of an event E is

$$P(E) = \frac{n(E)}{n(S)}.$$

In this problem,

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}.$$

20. Let O be the event "getting an odd number."

$$O = \{1, 3, 5\}$$
$$P(O) = \frac{3}{6} = \frac{1}{2}$$

21. "Getting a number less than 5" is the event $E = \{1, 2, 3, 4\}$, so $n(E) = 4$.

$$P(E) = \frac{4}{6} = \frac{2}{3}.$$

22. Let A be the event "getting a number greater than 2."

$$A = \{3, 4, 5, 6\}$$
$$P(A) = \frac{4}{6} = \frac{2}{3}$$

23. "Getting a 3 or a 4" is the event $E = \{3, 4\}$, so $n(E) = 2$.

$$P(E) = \frac{2}{6} = \frac{1}{3}.$$

24. Let B be the event "getting any number except 3."

$$B = \{1, 2, 4, 5, 6\}$$
$$P(B) = \frac{5}{6}$$

For Exercises 25–34, the sample space contains all 52 cards in the deck, so $n(S) = 52$.

25. Let E be the event "a 9 is drawn." There are four 9's in the deck, so $n(E) = 4$.

$$P(9) = P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

26. Let B be the event "drawing a black card." There are 26 black cards in the deck, 13 spades and 13 clubs.

$$n(B) = 26$$
$$P(B) = \frac{26}{52} = \frac{1}{2}$$

27. Let F be the event "a black 9 is drawn." There are two black 9's in the deck, so $n(F) = 2$.

$$P(\text{black } 9) = P(F) = \frac{n(F)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

28. Let H be the event "a heart is drawn." There are 13 hearts in the deck.

$$n(H) = 13$$
$$P(H) = \frac{13}{52} = \frac{1}{4}$$

29. Let G be the event "a 9 of hearts is drawn." There is only one 9 of hearts in a deck of 52 cards, so $n(G) = 1$.

$$P(9 \text{ of hearts}) = P(G) = \frac{n(G)}{n(S)} = \frac{1}{52}$$

30. Let F be the event "drawing a face card." The face cards are the jack, queen, and king of each of the four suits.

$$n(F) = 12$$
$$P(F) = \frac{12}{52} = \frac{3}{13}$$

31. Let H be the event "a 2 or a queen is drawn." There are four 2's and four queens in the deck, so $n(H) = 8$.

$$P(2 \text{ or queen}) = P(H) = \frac{n(H)}{n(S)} = \frac{8}{52} = \frac{2}{13}$$

32. Let R be the event "drawing a black 7 or a red 8." There are two black 7's and two red 8's.

$$n(R) = 4$$
$$P(R) = \frac{4}{52} = \frac{1}{13}$$

33. Let E be the event "a red card or a ten is drawn." There are 26 red cards and 4 tens in the deck. But 2 tens are red cards and are counted twice. Use the result from the previous section.

$$\begin{aligned} n(E) &= n(\text{red cards}) + n(\text{tens}) - n(\text{red tens}) \\ &= 26 + 4 - 2 \\ &= 28 \end{aligned}$$

Now calculate the probability of E.

$$P(\text{red cards or ten}) = \frac{n(E)}{n(S)} = \frac{28}{52} = \frac{7}{13}$$

34. Let E be the event "a spade or a king is drawn." There are 13 spades and 4 kings in the deck. But the king of spades is counted twice. Use the result from the previous section.

$$n(E) = n(\text{spades}) + n(\text{kings}) - n(\text{King of spades}) = 13 + 4 - 1 = 16$$

Now calculate the probability of E.

$$P(\text{spade or king}) = \frac{n(E)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

For Exercises 35–40**,** the sample space consists of all the marbles in the jar. There are $3 + 4 + 5 + 8 = 20$ marbles, so $n(S) = 20$.

35. 3 of the marbles are white, so

$$P(\text{white}) = \frac{3}{20}.$$

36. There are 4 orange marbles.

$$P(\text{orange}) = \frac{4}{20} = \frac{1}{5}$$

37. 5 of the marbles are yellow, so

$$P(\text{yellow}) = \frac{5}{20} = \frac{1}{4}.$$

38. There are 8 black marbles.

$$P(\text{black}) = \frac{8}{20} = \frac{2}{5}$$

39. $3 + 4 + 5 = 12$ of the marbles are not black, so

$$P(\text{not black}) = \frac{12}{20} = \frac{3}{5}.$$

40. There are 9 marbles which are orange or yellow.

$$P(\text{orange or yellow}) = \frac{9}{20}$$

41. It is possible to establish an exact probability for this event, so it is not an empirical probability.

42. This is empirical; only a survey could determine the probability.

43. It is not possible to establish an exact probability for this event, so this an empirical probability.

44. This is not empirical; a formula can compute the probability exactly.

45. It is not possible to establish an exact probability for this event, so this is an empirical probability.

46. This is empirical, based on experience and conditions rather than probability theory.

47. The gambler's claim is a mathematical fact, so this is not an empirical probability.

48. This is empirical, based on experience rather than probability theory.

49. The outcomes are not equally likely.

50. Let W_1 be the event "win on the first draw" and W_2 be the event "win on the second draw."

First of all, $P(W_1) = \frac{1}{3}$, a simple choice from three slips of paper.

To determine $P(W_2)$, the first slip of paper is drawn (from the three available) and thrown away. There are two slips left which may be chosen in any way. So $P(W_2) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$. Altogether, the probability of winning using this strategy is

$$P(W_1) + P(W_2) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}.$$

51. E: worker is female

F: worker has worked less than 5 yr

G: worker contributes to a voluntary retirement plan

(a) E' occurs when E does not, so E' is the event "worker is male."

(b) $E \cap F$ occurs when both E and F occur, so $E \cap F$ is the event "worker is female and has worked less than 5 yr."

(c) $E \cup G'$ is the event "worker is female or does not contribute to a voluntary retirement plan."

(d) F' occurs when F does not, so F' is the event "worker has worked 5 yr or more."

(e) $F \cup G$ occurs when F or G occurs or both, so $F \cup G$ is the event "worker has worked less than 5 yr or contributes to a voluntary retirement plan."

(f) $F' \cap G'$ occurs when F does not and G does not, so $F' \cap G'$ is the event "worker has worked 5 yr or more and does not contribute to a voluntary retirement plan."

52. **(a)** $P(\text{Federal government}) = \dfrac{103.7}{397.6} = 0.2608$

(b) $P(\text{Industry}) = \dfrac{267.8}{397.6} = 0.6735$

(c) $P(\text{Academic institutions}) = \dfrac{10.6}{397.6} = 0.0267$

53. **(a)** From the solution to Exercise 42 in Section 7.2 we know that 80 investors made all three types of investments, so the probability that a randomly chosen professor invested in stocks and bonds and certificates of deposit is $\frac{80}{150} = \frac{8}{15}$.

(b) From the solution to Exercise 42 in Section 7.2 we find that $80 - 65$ or 15 professors invested only in bonds, so the probability that a randomly chosen professor invested only in bonds is $\frac{15}{150} = \frac{1}{10}$.

54. **(a)** $P\left(\begin{matrix}\text{civilian laborer in} \\ \text{2008 is 55 or older}\end{matrix}\right) = \dfrac{27.9}{154.3} = 0.1808$

(b) $P\left(\begin{matrix}\text{civilian laborer in} \\ \text{2018 is 55 or older}\end{matrix}\right) = \dfrac{39.8}{166.8} = 0.2386$

55. E: person smokes

F: person has a family history of heart disease

G: person is overweight

(a) G': "person is not overweight."

(b) $F \cap G$: "person has a family history of heart disease and is overweight."

(c) $E \cup G'$: "person smokes or is not overweight."

56. E: person smokes

F: person has a family history of heart disease

G: person is overweight

(a) $E \cup F$ occurs when E or F or both occur, so $E \cup F$ is the event "person smokes or has a family history of heart disease, or both."

(b) $E' \cap F$ occurs when E does not occur and F does occur, so $E' \cap F$ is the event "person does not smoke and has a family history of heart disease."

(c) $F' \cup G'$ is the event "person does not have a family history of heart disease or is not overweight, or both."

57. **(a)** $P(\text{heart disease}) = \dfrac{615{,}651}{2{,}424{,}059} = 0.2540$

(b) $P(\text{cancer or heart disease}) = \dfrac{1{,}175{,}838}{2{,}424{,}059} = 0.4851$

(c) $P(\text{not accident and not diabetes mellitus})$

$$= \frac{2{,}424{,}059 - 117{,}075 - 70{,}905}{2{,}424{,}059} = \frac{2{,}236{,}079}{2{,}424{,}059} = 0.9225$$

58. The total population for 2020 is 322,742, and the total for 2050 is 393,931.

(a) $P(\text{Hispanic in 2020}) = \dfrac{52{,}652}{322{,}742} \approx 0.1631$

(b) $P(\text{Hispanic in 2050}) = \dfrac{96{,}508}{393{,}931} \approx 0.2450$

(c) $P(\text{Black in 2020}) = \dfrac{41{,}538}{322{,}742} \approx 0.1287$

(d) $P(\text{Black in 2050}) = \dfrac{53{,}555}{393{,}931} \approx 0.1360$

59. $P(\text{served 20–29 years}) = \dfrac{17}{100} = 0.17$

60. **(a)** $P(\text{Corps}) = \dfrac{9188}{91{,}950} \approx 0.0999$

(b) $P(\text{lost in battle}) = \dfrac{22{,}803}{91{,}950} \approx 0.2480$

(c) $P(\text{I Corps lost in battle}) = \dfrac{6059}{12{,}222} \approx 0.4957$

(d) $P(\text{I Corps not lost in battle})$

$$= \frac{12{,}222 - 6059}{12{,}222} \approx 0.5043$$

$P(\text{II Corps not lost in battle})$

$$= \frac{11{,}347 - 4369}{11{,}347} \approx 0.6150$$

$P(\text{III Corps not lost in battle})$

$$= \frac{10{,}675 - 4211}{10{,}675} \approx 0.6055$$

$P(\text{V Corps not lost in battle})$

$$= \frac{10{,}907 - 2187}{10{,}907} \approx 0.7995$$

$P(\text{VI Corps not lost in battle})$
$= \frac{13{,}596 - 242}{13{,}596} \approx 0.9822$

$P(\text{XI Corps not lost in battle})$
$= \frac{9188 - 3801}{9188} \approx 0.5863$

$P(\text{XII Corps not lost in battle})$
$= \frac{9788 - 1082}{9788} \approx 0.8895$

$P(\text{Cavalry not lost in battle})$
$= \frac{11{,}851 - 610}{11{,}851} \approx 0.9485$

$P(\text{Artillery not lost in battle})$
$= \frac{2376 - 242}{2376} \approx 0.8981$

VI Corps had the highest probability of not being lost in battle.

(e) $P(\text{I Corps loss}) = \frac{6059}{12{,}222} \approx 0.4957$

$P(\text{II Corps loss}) = \frac{4369}{11{,}347} \approx 0.3850$

$P(\text{III Corps loss}) = \frac{4211}{10{,}675} \approx 0.3945$

$P(\text{V Corps loss}) = \frac{2187}{10{,}907} \approx 0.2005$

$P(\text{VI Corps loss}) = \frac{242}{13{,}596} \approx 0.0178$

$P(\text{XI Corps loss}) = \frac{3801}{9188} \approx 0.4137$

$P(\text{XII Corps loss}) = \frac{1082}{9788} \approx 0.1105$

$P(\text{Cavalry loss}) = \frac{610}{11{,}851} \approx 0.0515$

$P(\text{Artillery loss}) = \frac{242}{2376} \approx 0.1019$

I Corps had the highest probability of loss.

61. (a) $P(\text{III Corps}) = \frac{22{,}083}{70{,}076} \approx 0.3151$

(b) $P(\text{lost in battle}) = \frac{22{,}557}{70{,}076} \approx 0.3219$

(c) $P(\text{I Corps lost in battle}) = \frac{7661}{20{,}706} \approx 0.3700$

(d) $P(\text{I Corps not lost in battle})$
$= \frac{20{,}706 - 7661}{20{,}706} \approx 0.6300$

$P(\text{II Corps not lost in battle})$
$= \frac{20{,}666 - 6603}{20{,}666} \approx 0.6805$

$P(\text{III Corps not lost in battle})$
$= \frac{22{,}083 - 8007}{22{,}083} \approx 0.6374$

$P(\text{Cavalry not lost in battle})$
$= \frac{6621 - 286}{6621} \approx 0.9568$

The Cavalry had the highest probability of not being lost in battle.

(e) $P(\text{I Corps loss}) = \frac{7661}{20{,}706} \approx 0.3700$

$P(\text{II Corps loss}) = \frac{6603}{20{,}666} \approx 0.3195$

$P(\text{III Corps loss}) = \frac{8007}{22{,}083} \approx 0.3626$

$P(\text{Cavalry loss}) = \frac{286}{6621} \approx 0.0432$

I Corps had the highest probability of loss.

62. (a) $P(\text{brought costumes and food}) = \frac{20}{75} = \frac{4}{15}$

(b) $P(\text{brought crafts, but neither food nor costumes}) = \frac{4}{75}$

(c) $P(\text{brought food or costumes}) = \frac{61}{75}$

63. There were 342 in attendance.

(a) $P(\text{speaks Cantonese}) = \frac{150}{342} = \frac{25}{57}$

(b) $P(\text{does not speaks Cantonese}) = \frac{192}{342} = \frac{32}{57}$

(c) $P(\text{woman who did not light firecracker})$.
$= \frac{72}{342} = \frac{4}{19}$

7.4 Basic Concepts of Probability

Your Turn 1

Let A stand for the event "ace" and C stand for the event "club."

$$\begin{aligned} P(A \cup C) &= P(A) + P(C) - P(A \cap C) \\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\ &= \frac{16}{52} = \frac{4}{13} \end{aligned}$$

Your Turn 2

Let E stand for the event "eight" and B stand for the event "both dice show the same number." From Figure 18 we see that E contains the 5 events 6-2, 5-3, 4-4, 3-5, and 2-6. B contains the 6 events 1-1, 2-2, 3-3, 4-4, 5-5, and 6-6. Only the even 4-4 belongs to both E and B. The sample space contains 36 equally likely events.

$$\begin{aligned} P(E \cup B) &= P(E) + P(B) - P(E \cap B) \\ &= \frac{5}{36} + \frac{6}{36} - \frac{1}{36} \\ &= \frac{10}{36} = \frac{5}{18} \end{aligned}$$

Your Turn 3

The complement of the event "sum < 11" is "sum $= 11$ or sum $= 12$." Figure 18 shows that "sum $= 11$ or sum $= 12$" contains the 3 events 5-6, 6-5, and 6-6.

$$\begin{aligned} P(\text{sum} < 11) &= 1 - P(\text{sum} = 11 \text{ or sum} = 12) \\ &= 1 - \frac{3}{36} \\ &= \frac{33}{36} = \frac{11}{12} \end{aligned}$$

Your Turn 4

Let E be the event "snow tomorrow." Since $P(E) = \frac{3}{10}$, $P(E') = 1 - P(E) = \frac{7}{10}$. The odds in favor of snow tomorrow are

$$\frac{P(E)}{P(E')} = \frac{3/10}{7/10} = \frac{3}{7}.$$

We can write these odds as 3 to 7 or 3:7.

Your Turn 5

Let E be the event "package delivered on time." The odds in favor of E are 17 to 3, so $P(E') = \frac{3}{17+3} = \frac{3}{20}$. The probability that the package will not be delivered on time is $\frac{3}{20}$.

Your Turn 6

If the odds against the horse winning are 7 to 3,

$P(\text{loses}) = \frac{7}{7+3} = \frac{7}{10}$, so $P(\text{wins}) = 1 - \frac{7}{10} = \frac{3}{10}$.

7.4 Exercises

3. A person can own a dog and own an MP3 player at the same time. No, these events are not mutually exclusive.

4. A person can be from Texas and be a business major at the same time. No, these events are not mutually exclusive.

5. A person can be retired and be over 70 years old at the same time. No, these events are not mutually exclusive.

6. A person cannot be a teenager and be 70 years old at the same time. Yes, these events are mutually exclusive.

7. A person cannot be one of the ten tallest people in the United States and be under 4 feet tall at the same time. Yes, these events are mutually exclusive.

8. A person can be male and be a nurse at the same time. No, these events are not mutually exclusive.

9. When two dice are rolled, there are 36 equally likely outcomes.

(a) Of the 36 ordered pairs, there is only one for which the sum is 2, namely $\{(1,1)\}$. Thus,

$$P(\text{sum is } 2) = \frac{1}{36}.$$

(b) $\{(1,3),(2,2),(3,1)\}$ comprise the ways of getting a sum of 4. Thus,

$$P(\text{sum is } 4) = \frac{3}{36} = \frac{1}{12}.$$

(c) $\{(1,4),(2,3),(3,2),(4,1)\}$ comprise the ways of getting a sum of 5. Thus,

$$P(\text{sum is } 5) = \frac{4}{36} = \frac{1}{9}.$$

(d) $\{(1,5),(2,4),(3,3),(4,2),(5,1)\}$ comprise the ways of getting a sum of 6. Thus,

$$P(\text{sum is } 6) = \frac{5}{36}.$$

10. When the two dice are rolled, there are 36 equally likely outcomes. Let 5-3 represent the outcome "the first die shows a 5 and the second die shows a 3," and so on.

(a) Rolling a sum of 8 occurs when the outcome is 2-6, 3-5, 4-4, 5-3, or 6-2. Therefore, since there are five such outcomes, the probability of this event is

$$P(\text{sum is } 8) = \frac{5}{36}.$$

(b) A sum of 9 occurs when the outcome is 3-6, 4-5, 5-4, or 6-3, so

$$P(\text{sum is } 9) = \frac{4}{36} = \frac{1}{9}.$$

(c) A sum of 10 occurs when the outcome is 4-6, 5-5, or 6-4, so

$$P(\text{sum is } 10) = \frac{3}{36} = \frac{1}{12}.$$

(d) A sum of 13 does not occur in any of the 36 outcomes, so

$$P(\text{sum is } 13) = \frac{0}{36} = 0.$$

11. Again, when two dice are rolled there are 36 equally likely outcomes.

(a) Here, the event is the union of four mutually exclusive events, namely, the sum is 9, the sum is 10, the sum is 11, and the sum is 12. Hence,

$$\begin{aligned} P(\text{sum is 9 or more}) &= P(\text{sum is } 9) + P(\text{sum is } 10) \\ &\quad + P(\text{sum is } 11) + (\text{sum is } 12) \\ &= \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} \\ &= \frac{10}{36} = \frac{5}{18}. \end{aligned}$$

(b) $P(\text{sum is less than } 7)$

$$\begin{aligned} &= P(2) + P(3) + P(4) + P(5) + P(6) \\ &= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36} \\ &= \frac{15}{36} \\ &= \frac{5}{12} \end{aligned}$$

(c) $P(\text{sum is between 5 and } 8)$

$$\begin{aligned} &= P(\text{sum is } 6) + P(\text{sum is } 7) \\ &= \frac{5}{36} + \frac{6}{36} \\ &= \frac{11}{36} \end{aligned}$$

12. Again, when two dice are rolled, there are 36 equally likely outcomes.

(a) $P(\text{sum is not more than } 5)$

$$\begin{aligned} &= P(\text{sum is } 1) + P(\text{sum is } 2) \\ &\quad + P(\text{sum is } 3) + P(\text{sum is } 4) \\ &\quad + P(\text{sum is } 5) \\ &= \frac{0}{36} + \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} \\ &= \frac{10}{36} = \frac{5}{18} \end{aligned}$$

(b) $P(\text{sum is not less than } 8)$

$$\begin{aligned} &= P(8) + P(9) + P(10) \\ &\quad + P(11) + P(12) \\ &= \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36} + \frac{1}{36} \\ &= \frac{15}{36} \\ &= \frac{5}{12} \end{aligned}$$

(c) $P(\text{sum is between 3 and } 7)$

$$\begin{aligned} &= P(4) + P(5) + P(6) \\ &= \frac{3}{36} + \frac{4}{36} + \frac{5}{36} \\ &= \frac{12}{36} \\ &= \frac{1}{3} \end{aligned}$$

13. $P(\text{first die is 3 or sum is } 8)$

$$\begin{aligned} &= P(\text{first die is } 3) + P(\text{sum is } 8) \\ &\quad - P(\text{first die is 3 and sum is } 8) \\ &= \frac{6}{36} + \frac{5}{36} - \frac{1}{36} \\ &= \frac{10}{36} \\ &= \frac{5}{18} \end{aligned}$$

14. $P(\text{second die is 5 or the sum is 10})$

$$= P(\text{second die is 5}) + P(\text{sum is 10}) - P(\text{second die is 5 and sum is 10})$$
$$= \frac{6}{36} + \frac{3}{36} - \frac{1}{36}$$
$$= \frac{8}{36}$$
$$= \frac{2}{9}$$

15. **(a)** The events E, "9 is drawn," and F, "10 is drawn," are mutually exclusive, so $P(E \cap F) = 0.$ Using the union rule,

$$P(9 \text{ or } 10) = P(9) + P(10) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}.$$

(b) $$P(\text{red or } 3) = P(\text{red}) + P(3) - P(\text{red and } 3) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

(c) Since these events are mutually exclusive,

$$P(9 \text{ or black } 10) = P(9) + P(\text{black } 10) = \frac{4}{52} + \frac{2}{52} = \frac{6}{52} = \frac{3}{26}.$$

(d) $$P(\text{heart or black}) = \frac{13}{52} + \frac{26}{52} = \frac{39}{52} = \frac{3}{4}.$$

(e) $P(\text{face card or diamond})$

$$= P(\text{face card}) + P(\text{diamond}) - P(\text{face card and diamond})$$
$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52}$$
$$= \frac{22}{52}$$
$$= \frac{11}{26}$$

16. **(a)** Less than a 4 would be an ace, a 2, or a 3. There are a total of 12 aces, 2's, and 3's in a deck of 52, so

$$P(\text{ace or 2 or 3}) = \frac{12}{52} = \frac{3}{13}.$$

(b) There are 13 diamonds plus three 7's in other suits, so

$$P(\text{diamond or 7}) = \frac{16}{52} = \frac{4}{13}.$$

Alternatively, using the union rule for probability,

$$P(\text{diamond}) + P(7) - P(7 \text{ of diamonds}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

(c) There are 26 black cards plus 2 red aces, so

$$P(\text{black or ace}) = \frac{28}{52} = \frac{7}{13}.$$

(d) $$P(\text{heart or jack}) = P(\text{heart}) + P(\text{jack}) - P(\text{heart and jack}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

(e) There are 26 red cards plus 6 black face cards, so

$$P(\text{red or face card}) = \frac{32}{52} = \frac{8}{13}.$$

17. **(a)** Since these events are mutually exclusive,

$$P(\text{brother or uncle}) = P(\text{brother}) + P(\text{uncle}) = \frac{2}{13} + \frac{3}{13} = \frac{5}{13}.$$

(b) Since these events are mutually exclusive,

$$P(\text{brother or cousin}) = P(\text{brother}) + P(\text{cousin}) = \frac{2}{13} + \frac{5}{13} = \frac{7}{13}.$$

(c) Since these events are mutually exclusive,

$$P(\text{brother or mother}) = P(\text{brother}) + P(\text{mother}) = \frac{2}{13} + \frac{1}{13} = \frac{3}{13}.$$

18. **(a)** There are 3 uncles plus 5 cousins out of 13, so

$$P(\text{uncle or cousin}) = \frac{8}{13}.$$

(b) There are 3 uncles, 2 brothers, and 5 cousins, for a total of 10 out of 13, so

$$P(\text{male or cousin}) = \frac{10}{13}.$$

(c) There are 2 aunts, 5 cousins, and 1 mother, for a total of 5 out of 10, so

$$P(\text{female or cousin}) = \frac{8}{13}.$$

19. **(a)** There are 5 possible numbers on the first slip drawn, and for each of these, 4 possible numbers on the second, so the sample space contains $5 \cdot 4 = 20$ ordered pairs. Two of these ordered pairs have a sum of 9: (4, 5) and (5, 4). Thus,

$$P(\text{sum is } 9) = \frac{2}{20} = \frac{1}{10}.$$

(b) The outcomes for which the sum is 5 or less are (1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (3, 1), (3, 2), and (4,1). Thus,

$$P(\text{sum is 5 or less}) = \frac{8}{20} = \frac{2}{5}.$$

(c) Let A be the event "the first number is 2" and B the event "the sum is 6." Use the union rule.

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{4}{20} + \frac{4}{20} - \frac{1}{20} \\ &= \frac{7}{20} \end{aligned}$$

20. **(a)** The sample space for this experiment is listed in part (b) below. The only outcomes in which both numbers are even are (2, 4) and (4, 2), so

$$P(\text{even}) = \frac{2}{20} = \frac{1}{10}.$$

(b) The sample space is

$$\{(\underline{1,2}), (1,3), (\underline{1,4}), (\underline{1,5}), (\underline{2,1}), (\underline{2,3}), (\underline{2,4}), (\underline{2,5}), (3,1), (\underline{3,2}),$$
$$(\underline{3,4}), (\underline{3,5}), (\underline{4,1}), (\underline{4,2}), (\underline{4,3}), (\underline{4,5}), (\underline{5,1}), (\underline{5,2}), (\underline{5,3}), (\underline{5,4})\}.$$

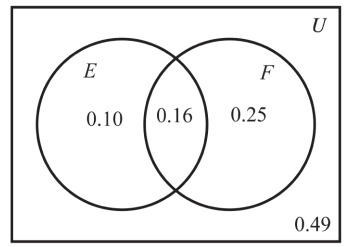
The 18 underlined pairs are the outcomes in which one number is even or greater than 3, so

$$P(\text{even or} > 3) = \frac{18}{20} = \frac{9}{10}.$$

(c) The sum is 5 in the outcomes (1, 4), (2, 3), (3, 2), and (4, l). The second draw is 2 in the outcomes (1, 2), (3, 2), (4, 2), and (5, 2). There are 7 distinct outcomes out of 20, so

$$P(\text{sum is 5 or second number is 2}) = \frac{7}{20}.$$

21. Since $P(E \cap F) = 0.16,$ the overlapping region $E \cap F$ is assigned the probability 0.16 in the diagram. Since $P(E) = 0.26$ and $P(E \cap F) = 0.16,$ the region in E but not F is given the label 0.10. Similarly, the remaining regions are labeled.



(a) $$\begin{aligned} P(E \cup F) &= 0.10 + 0.16 + 0.25 \\ &= 0.51 \end{aligned}$$

Consequently, the part of U outside $E \cup F$ receives the label

$$1 - 0.51 = 0.49.$$

(b) $$\begin{aligned} P(E' \cap F) &= P(\text{in } F \text{ but not in } E) \\ &= 0.25 \end{aligned}$$

(c) The region $E \cap F'$ is that part of E which is not in F. Thus,

$$P(E \cap F') = 0.10.$$

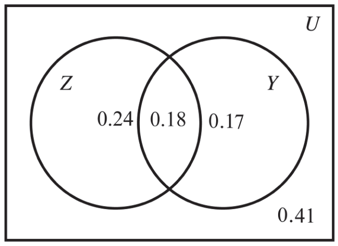
(d) $$\begin{aligned} P(E' \cup F') &= P(E') + P(F') - P(E' \cap F') \\ &= 0.74 + 0.59 - 0.49 \\ &= 0.84 \end{aligned}$$

22. $P(Z) = 0.42, P(Y) = 0.35, P(Z \cup Y) = 0.59$

Begin by using the union rule for probability.

$$\begin{aligned} P(Z \cup Y) &= P(Z) + P(Y) - P(Z \cap Y) \\ 0.59 &= 0.42 + 0.35 - P(Z \cap Y) \\ 0.59 &= 0.77 - P(Z \cap Y) \\ -0.18 &= -P(Z \cap Y) \\ 0.18 &= P(Z \cap Y) \end{aligned}$$

This gives the first value to be labeled in the Venn diagram. Then the part of Z outside Y must contain $0.42 - 0.18 = 0.24,$ and the part of Y outside Z must contain $0.35 - 0.18 = 0.17.$ Observe that $0.24 + 0.18 + 0.17 = 0.59,$ which agrees with the given information that $P(Z \cup Y) = 0.59.$ The part of U outside both Y and Z must contain $1 - 0.59 = 0.41.$



This Venn diagram may now be used to find the following probabilities.

(a) $Z' \cap Y'$ is the event presented by the part of the Venn diagram that is outside Z and outside Y.

$$P(Z' \cap Y') = 0.41$$

(b) $Z' \cup Y'$ is everything outside Z or outside Y or both, which is all of U except $Z \cap Y$.

$$P(Z' \cap Y') = 1 - 0.18 = 0.82$$

(c) $Z' \cup Y$ is everything outside Z or inside Y or both.

$$P(Z' \cup Y) = 0.17 + 0.14 + 0.18 = 0.76$$

(d) $Z \cap Y'$ is everything inside Z and outside Y.

$$P(Z \cap Y') = 0.24$$

23. **(a)** The sample space is

3 – 1	3 – 1	3 – 5	3 – 5	3 – 9	3 – 9
3 – 1	3 – 1	3 – 5	3 – 5	3 – 9	3 – 9
4 – 1	4 – 1	4 – 5	4 – 5	4 – 9	4 – 9
4 – 1	4 – 1	4 – 5	4 – 5	4 – 9	4 – 9
8 – 1	8 – 1	8 – 5	8 – 5	8 – 9	8 – 9
8 – 1	8 – 1	8 – 5	8 – 5	8 – 9	8 – 9

where the first number in each pair is the number that appears on A and the second the number that appears on B. B beats A in 20 of 36 possible outcomes. Thus,

$$P(B \text{ beats } A) = \frac{20}{36} = \frac{5}{9}.$$

(b) The sample space is

1 – 2	1 – 2	1 – 6	1 – 6	1 – 7	1 – 7
1 – 2	1 – 2	1 – 6	1 – 6	1 – 7	1 – 7
5 – 2	5 – 2	5 – 6	5 – 6	5 – 7	5 – 7
5 – 2	5 – 2	5 – 6	5 – 6	5 – 7	5 – 7
9 – 2	9 – 2	9 – 6	9 – 6	9 – 7	9 – 7
9 – 2	9 – 2	9 – 6	9 – 6	9 – 7	9 – 7

where the first number in each pair is the number that appears on B and the second the number that appears on C. C beats B in 20 of 36 possible outcomes. Thus,

$$P(C \text{ beats } B) = \frac{20}{36} = \frac{5}{9}.$$

(c) The sample space is

3 – 2	3 – 2	3 – 6	3 – 6	3 – 7	3 – 7
3 – 2	3 – 2	3 – 6	3 – 6	3 – 7	3 – 7
4 – 2	4 – 2	4 – 6	4 – 6	4 – 7	4 – 7
4 – 2	4 – 2	4 – 6	4 – 6	4 – 7	4 – 7
8 – 2	8 – 2	8 – 6	8 – 6	8 – 7	8 – 7
8 – 2	8 – 2	8 – 6	8 – 6	8 – 7	8 – 7

where the first number in each pair is the number that appears on A and the second the number that appears on C. A beats C in 20 of 36 possible outcomes. Thus,

$$P(A \text{ beats } C) = \frac{20}{36} = \frac{5}{9}.$$

24. The statement is not correct.

Assume two dice are rolled.

$$\begin{aligned} P(\text{you win}) &= P(\text{first die is greater than second}) \\ &= \frac{15}{36} \\ &= \frac{5}{12} \end{aligned}$$

$$\begin{aligned} P(\text{other player wins}) &= 1 - P(\text{you win}) \\ &= 1 - \frac{5}{12} \\ &= \frac{7}{12} \end{aligned}$$

27. Let E be the event "a 3 is rolled."

$$P(E) = \frac{1}{6} \text{ and } P(E') = \frac{5}{6}.$$

The odds in favor of rolling a 3 are

$$\frac{P(E)}{P(E')} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5},$$

which is written "1 to 5."

28. Let E be the event "4, 5, or 6 is rolled." Here $E = \{4,5,6\}$, so $P(E) = \frac{3}{6} = \frac{1}{2}$, and $P(E') = \frac{1}{2}$. The odds in favor of rolling 4, 5 or 6 are

$$\frac{P(E)}{P(E')} = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{1}.$$

which is written "1 to 1."

29. Let E be the event "a 2, 3, 4, or 5 is rolled." Here $P(E) = \frac{4}{6} = \frac{2}{3}$ and $P(E') = \frac{1}{3}$. The odds in favor of E are

$$\frac{P(E)}{P(E')} = \frac{\frac{2}{3}}{\frac{1}{3}} = \frac{2}{1},$$

which is written "2 to 1."

30. Let F be the event "some number less than 6 is rolled." Here $F = \{1, 2, 3, 4, 5\}$, so $P(F) = \frac{5}{6}$ and $P(F') = \frac{1}{6}$. The odds in favor of rolling a number less than 2 are

$$\frac{P(F)}{P(F')} = \frac{\frac{5}{6}}{\frac{1}{6}} = \frac{5}{1},$$

which is written "5 to 1."

31. **(a)** Yellow: There are 3 ways to win and 15 ways to lose. The odds in favor of drawing yellow are 3 to 15, or 1 to 5.

(b) Blue: There are 11 ways to win and 7 ways to lose; the odds in favor of drawing blue are 11 to 7.

(c) White: There are 4 ways to win and 14 ways to lose; the odds in favor of drawing white are 4 to 14, or 2 to 7.

(d) Not white: Since the odds in favor of white are 2 to 7, the odds in favor of not white are 7 to 2.

32. **(a)** From Figure 18 we see that a sum of 3 includes the 2 outcomes 1-2 and 2-1; there are $36 - 2$ or 34 remaining outcomes. Thus the odds in favor of a sum of 3 are 2 to 34 or 1 to 17.

(b) From Figure 18 we see that a sum of 7 or 11 includes the 8 outcomes 1-6, 2-5, 3-4, 4-3, 5-2, 6-1, 5-6, and 6-5; there are $36 - 8$ or 28 remaining outcomes. Thus the odds in favor of a sum of 7 or 11 are 8 to 28 or 2 to 7.

(c) A sum less than 5 must be 2, 3 or 4, which includes the 6 outcomes 1-1, 1-2, 2-1, 1-3, 2-2, and 3-1; there are $36 - 6 = 30$ remaining outcomes. Thus the odds in favor of a sum less than 5 are 6 to 30 or 1 to 5.

(d) Not a sum of 6 is everything except the 5 outcomes 1-5, 2-4, 3-3, 4-2, and 5-1. So $36 - 5 = 31$ outcomes favor the event and the odds in favor of "not a sum of 6" are 31 to 5.

35. Each of the probabilities is between 0 and 1 and the sum of all the probabilities is

$$0.09 + 0.32 + 0.21 + 0.25 + 0.13 = 1,$$

so this assignment is possible.

36. The probability assignment is possible because the probability of each outcome is a number between 0 and 1, and the sum of the probabilities of all the outcomes is

$$0.92 + 0.03 + 0 + 0.02 + 0.03 = 1.$$

37. The sum of the probabilities

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = \frac{117}{120} < 1,$$

so this assignment is not possible.

38. The probability assignment is not possible. All of the probabilities are between 0 and 1, but the sum of the probabilities is

$$\frac{1}{5} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10} = \frac{13}{12}$$

which is greater than 1.

39. This assignment is not possible because one of the probabilities is -0.08, which is not between 0 and 1. A probability cannot be negative.

40. The probability assignment is not possible. One of the probabilities is negative instead of being between 0 and 1, and the sum of the probabilities is not 1.

41. The answers that are given are theoretical. Using the Monte Carlo method with at least 50 repetitions on a graphing calculator should give values close to these.

(a) 0.2778

(b) 0.4167

42. The answers that are given are theoretical. Using the Monte Carlo method with at least 50 repetitions on a graphing calculator should give values close to these.

(a) 0.2778

(b) 0.4167

43. The answers that are given are theoretical. Using the Monte Carlo method with at least 100 repetitions should give values close to these.

(a) 0.0463

(b) 0.2963

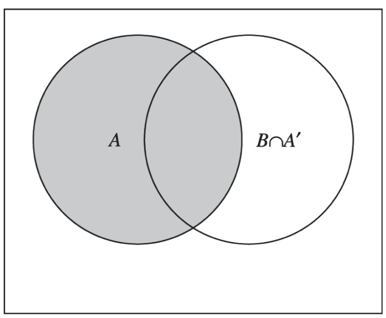
44. The answers that are given are theoretical. Using the Monte Carlo method with at least 50 repetitions on a graphing calculator should give values close to these.

(a) 0.15625

(b) 0.3125

46. First, notice $(B \cap A') \cup (A \cup B') = U$. As a result,

$$\begin{aligned} P(B \cap A') + (A \cup B') &= P(U) = 1 \\ P(B \cap A') &= 1 - P(A \cup B') \\ &= 1 - 0.9 = 0.1. \end{aligned}$$



From the Venn diagram, note that $A \cup B = A \cup (B \cap A')$. Therefore,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B \cap A') \\ 0.7 &= P(A) + 0.1 \\ P(A) &= 0.6 \end{aligned}$$

The correct answer is choice **d**.

47. Let C be the event "the calculator has a good case," and let B be the event "the calculator has good batteries."

$$\begin{aligned} &P(C \cap B) \\ &= 1 - P[(C \cap B)'] \\ &= 1 - P(C' \cup B') \\ &= 1 - [P(C') + P(B') - P(C' \cap B')] \\ &= 1 - (0.08 + 0.11 - 0.03) \\ &= 0.84 \end{aligned}$$

Thus, the probability that the calculator has a good case and good batteries is 0.84.

48. **(a)** $$\begin{aligned} P(\text{no profit}) &= 1 - P(\text{Profit}) \\ &= 1 - 0.74 \\ &= 0.26 \end{aligned}$$

(b) The odds against profit are $\frac{0.26}{0.74}$ or 13 to 37.

49. **(a)** $$\begin{aligned} P(\$500 \text{ or more}) &= 1 - P(\text{less than } \$500) \\ &= 1 - (0.21 + 0.17) \\ &= 1 - 0.38 \\ &= 0.62 \end{aligned}$$

(b) $$\begin{aligned} P(\text{less than } \$1000) &= 0.21 + 0.17 + 0.16 \\ &= 0.54 \end{aligned}$$

(c) $$\begin{aligned} P(\$500 \text{ to } \$2999) &= 0.16 + 0.15 + 0.12 \\ &= 0.43 \end{aligned}$$

(d) $$\begin{aligned} P(\$3000 \text{ or more}) &= 0.08 + 0.07 + 0.04 \\ &= 0.19 \end{aligned}$$

50. **(a)** $$\begin{aligned} P(\text{sales or service}) &= 0.2024 + 0.1015 \\ &= 0.3039 \end{aligned}$$

(b) $$\begin{aligned} P(\text{not in construction}) &= 1 - 0.0531 \\ &= 0.9469 \end{aligned}$$

(c) The probability of a worker being in production is 0.0585, so the probability of a worker not being in production is $1 - 0.0585 = 0.9415$. Thus the odds in favor of a worker being in production are $\frac{0.0585}{0.9415} = \frac{117}{1883} \approx 0.0621$ which we could write as 117 to 1883. Since $\frac{1}{16} = 0.0625$ these odds are approximately 1:16.

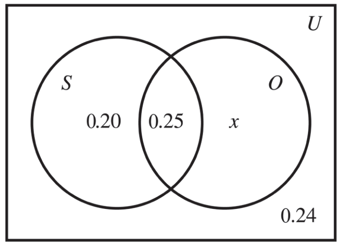
51. **(a)** The probability of Female and 16 to 24 years old is the entry in the first row in the Female column, which is 0.061.

(b) 16 to 54 years old includes the first two rows of the table, so adding the totals for these two rows we find that the probability is $0.127 + 0.634 = 0.761$.

(c) For Male or 25 to 54, we add the totals of the second row and the Male column and then subtract the value for Male and 25 to 54 in order not to count it twice. Thus the probability is $0.634 + 0.531 - 0.343 = 0.822$.

(d) For Female or 16 to 24 we add the totals of the first row and the Female column and then subtract the value for Female and 16 to 24 in order not to count it twice. Thus the probability is $0.127 + 0.469 - 0.061 = 0.535$.

52. Let S be the event "the person is short," and let O be the event "the person is overweight."



From the Venn diagram,

$$\begin{aligned} 0.20 + 0.25 + x + 0.24 &= 1 \\ 0.69 + x &= 1 \\ x &= 0.31. \end{aligned}$$

The probability that a person is

(a) overweight is $0.25 + 0.31 = 0.56$;

(b) short, but not overweight is 0.20;

(c) tall (not short) and overweight is 0.31.

53. $P(C) = 0.039, P(M \cap C) = 0.035,$
$P(M \cup C) = 0.491$

Place the given information in a Venn diagram by starting with 0.035 in the intersection of the regions for M and C.

Since $P(C) = 0.039, 0.039 - 0.035 = 0.004$

goes inside region C, but outside the intersection of C and M. Thus,

$$P(C \cap M') = 0.004.$$

Since

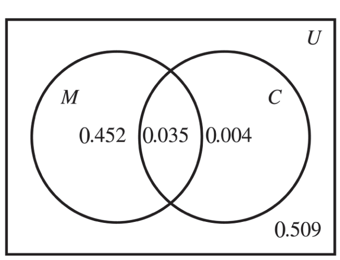
$$P(M \cup C) = 0.491, 0.491 - 0.035 - 0.004 = 0.452$$

goes inside region M, but outside the intersection of C and M. Thus, $P(M \cap C') = 0.452.$ The labeled regions have probability

$$0.452 + 0.035 + 0.004 = 0.491.$$

Since the entire region of the Venn diagram must have probability 1, the region outside M and C, or $M' \cap C'$, has probability

$$1 - 0.491 = 0.509.$$



(a) $P(C') = 1 - P(C)$
$= 1 - 0.039$
$= 0.961$

(b) $P(M) = 0.452 + 0.035$
$= 0.487$

(c) $P(M') = 1 - P(M)$
$= 1 - 0.487$
$= 0.513$

(d) $P(M' \cap C') = 0.509$

(e) $P(C \cap M') = 0.004$

(f) $P(C \cup M')$
$= P(C) + P(M') - P(C \cap M')$
$= 0.039 + 0.513 - 0.004$
$= 0.548$

54. **(a)** Since red is dominant, the event "plant has red flowers"

$$= \{RR, RW, WR\};\ P(\text{red}) = \frac{3}{4}.$$

(b) $P(\text{white}) = 1 - P(\text{red}) = \dfrac{1}{4}$

55. **(a)** Now red is no longer dominant, and RW or WR results in pink, so

$$P(\text{red}) = P(RR) = \frac{1}{4}.$$

(b) $P(\text{pink}) = P(RW) + P(WR)$
$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{1}{2}$

(c) $P(\text{white}) = P(WW) = \dfrac{1}{4}$

56. **(a)** $P(\text{no more than 4 good toes})$
$= 0.77 + 0.13 = 0.90$

(b) $P(\text{5 toes}) = 0.13 + 0.10 = 0.23$

57. Let L be the event "visit results in lab work" and R be the event "visit results in referral to specialist." We are given the probability a visit results in neither is 35%, so $P((L \cup R)') = 0.35.$ Since $P(L \cup R) = 1 - P((L \cup R)'),$ we have

$$P(L \cup R) = 1 - 0.35 = 0.65.$$

We are also given $P(L) = 0.40$ and $P(R) = 0.30.$ Using the union rule for probability,

$$\begin{aligned} P(L \cup R) &= P(L) + P(R) - P(L \cap R) \\ 0.65 &= 0.40 + 0.30 - P(L \cap R) \\ 0.65 &= 0.50 - P(L \cap R) \\ P(L \cap R) &= 0.05 \end{aligned}$$

The correct answer choice is **a**.

58. Let T be the event "patient visits a physical therapist" and C be the event "patient visits a chiropractor." If 22% of patients visit both a physical therapist and a chiropractor, then $P(T \cap C) = 0.22$. If 12% of patients visit neither, then $P((T \cup C)') = 0.12$. This means $P(T \cup C) = 1 - P((T \cup C)') = 1 - 0.12 = 0.88$.

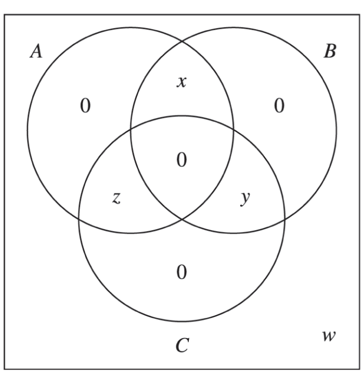
Let $x = P(T)$. If the probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist, then $P(C) = P(T) + 0.14 = x + 0.14$. Using the union rule for probability, we have

$$\begin{aligned} P(T \cup C) &= P(T) + P(C) - P(T \cap C) \\ 0.88 &= x + (x + 0.14) - 0.22 \\ 0.88 &= 2x - 0.08 \\ 0.96 &= 2x \\ x &= 0.48 \end{aligned}$$

The correct answer choice is **d**.

59. Let $x = P(A \cap B)$,
$y = P(B \cap C)$,
$z = P(A \cap C)$,
and $w = P((A \cup B \cup C)')$.

If an employee must choose exactly two or none of the supplementary coverages A, B, and C, then $P(A \cap B \cap C) = 0$ and the probabilities of the region representing a single choice of coverages A, B, or C are also 0. We can represent the choices and probabilities with the following Venn diagram.



The information given leads to the following system of equations.

$$\begin{aligned} x + y + z + w &= 1 \\ x \quad + z \quad &= \frac{1}{4} \\ x + y \quad &= \frac{1}{3} \\ y + z &= \frac{5}{12} \end{aligned}$$

Using a graphing calculator or computer program, the solution to the system is $x = 1/2$, $y = 1/4$, $z = 1/6$, $w = 1/2$. Since the probability that a randomly chosen employee will choose no supplementary coverage is w, the correct answer choice is **c**.

60. Gore: $\frac{1}{2+1} = \frac{1}{3}$

Daschle: $\frac{1}{4+1} = \frac{1}{5}$

Kerry: $\frac{1}{4+1} = \frac{1}{5}$

Dodd: $\frac{1}{4+1} = \frac{1}{5}$

Lieberman: $\frac{1}{5+1} = \frac{1}{6}$

Biden: $\frac{1}{5+1} = \frac{1}{6}$

Leahy: $\frac{1}{6+1} = \frac{1}{7}$

Feingold: $\frac{1}{8+1} = \frac{1}{9}$

Edwards: $\frac{1}{9+1} = \frac{1}{10}$

Gephardt: $\frac{1}{15+1} = \frac{1}{16}$

Find the sum of the probabilities.

$$\frac{1}{3} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{6} + \frac{1}{6} + \frac{1}{7} + \frac{1}{9} + \frac{1}{10} + \frac{1}{16}$$
$$= \frac{8483}{5040} \approx 1.68$$

Since the sum of the probabilities of all possible outcomes cannot be greater than 1, there is some thing wrong with the assignment of odds.

61. Since 55 of the workers were women, $130 - 55 = 75$ were men. Since 3 of the women earned more than \$40,000, $55 - 3 = 52$ of them earned \$40,000 or less. Since 62 of the men earned \$40,000 or less, $75 - 62 = 13$ earned more than \$40,000. These data for the 130 workers can be summarized in the following table.

	Men	Women
\$40,000 or less	62	52
Over \$40,000	13	3

(a) P(a woman earning \$40,000 or less)

$$= \frac{52}{130} = 0.4$$

(b) P(a man earning more than \$40,000)

$$= \frac{13}{130} = 0.1$$

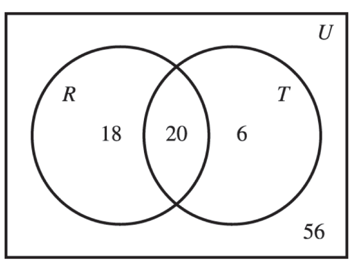
(c) P(a man or is earning more than \$40,000)

$$= \frac{62 + 13 + 3}{130}$$
$$= \frac{78}{130} = 0.6$$

(d) P(a woman or is earning \$40,000 or less)

$$= \frac{52 + 3 + 62}{130}$$
$$= \frac{117}{130} = 0.9$$

62. Let R be the event "the person buys rock music," and let T be the event "the person is a teenager."



The probability that a person

(a) is a teenager who buys nonrock music is $\frac{6}{100} = 0.06$;

(b) buys rock music or is a teenager is $\frac{18}{100} + \frac{20}{100} + \frac{6}{100} = 0.44$;

(c) is not a teenager is $1 - \frac{26}{100} = 0.74$;

(d) is not a teenager, but buys rock music, $\frac{18}{100} = 0.18$.

63. Let A be the set of refugees who came to escape abject poverty and B be the set of refugees who came to escape political oppression. Then $P(A) = 0.80$, $P(B) = 0.90$, and $P(A \cap B) = 0.70$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.80 + 0.90 - 0.70 = 1$$
$$P(A' \cap B') = 1 - P(A \cap B)$$
$$= 1 - 1 = 0$$

The probability that a refugee in the camp was neither poor nor seeking political asylum is 0.

64. There are 18 people, so $n(S) = 18$.

(a) P(Chinese)

$$= P(\text{Chinese man}) + P(\text{Chinese woman})$$
$$= \frac{3}{18} + \frac{4}{18}$$
$$= \frac{7}{18}$$

(b) P(Korean or woman)

$$= P(\text{Korean}) + P(\text{woman}) - P(\text{Korean woman})$$
$$= \frac{6}{18} + \frac{11}{18} - \frac{4}{18}$$
$$= \frac{13}{18}$$

(c) P(man or Vietnamese)

$$= P(\text{man}) + P(\text{Vietnamese}) - P(\text{Vietnamese man})$$
$$= \frac{7}{18} + \frac{5}{18} - \frac{2}{18}$$
$$= \frac{10}{18}$$
$$= \frac{5}{9}$$

(d) These events are mutually exclusive, so P(Chinese or Vietnamese)

$$= P(\text{Chinese}) + P(\text{Vietnamese})$$
$$= \frac{7}{18} + \frac{5}{18}$$
$$= \frac{12}{18}$$
$$= \frac{2}{3}.$$

(e) $P(\text{Korean woman}) = \frac{4}{18} = \frac{2}{9}$

65. The odds of winning are 3 to 2; this means there are 3 ways to win and 2 ways to lose, out of a total of $2 + 3 = 5$ ways altogether. Hence, the probability of losing is $\frac{2}{5}$.

66. **(a)** We divide each number in the given table by 203,375, which produces the following table of probabilities:

	A	*B*	*C*	*D*
O	0.0704	0.0595	0.0388	0.0059
E	0.2921	0.2555	0.2076	0.0578
M	0.0034	0.0045	0.0045	0

(b) $P(A) = 0.0704 + 0.2921 + 0.0034$
$= 0.3659$

(c) This is the sum of the entries in the *O* row and the *C* and *D* columns:

$$P(O \cap (C \cup D)) = 0.0388 + 0.0059 = 0.0447$$

(d) This is the sum of the entries in the *A* and *B* columns. We have already computed $P(A)$ in part (b).

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= 0.3659 + 0.0595 + 0.2555 + 0.0045 \\ &= 0.6854 \end{aligned}$$

(e) We can find $P(C \cup D)$ by subtracting the probability we found in (d) from 1:

$$\begin{aligned} P(C \cup D) &= 1 - 0.6854 \\ &= 0.3146 \end{aligned}$$

To this we want to add the first two values in the *E* row to form the union with *E*:

$$\begin{aligned} P(E \cup (C \cup D) &= 0.3146 + 0.2921 + 0.2555 \\ &= 0.8622 \end{aligned}$$

67. **(a)** P(somewhat or extremely intolerant of Facists)
$= P$(somewhat intolerant of Facists)
$+ P$(extremely intolerant of Facists)
$= \dfrac{27.1}{100} + \dfrac{59.5}{100} = \dfrac{86.6}{100} = 0.866$

(b) P(completely tolerant of Communists)
$= P$(no intolerance at all of Communists)
$= \dfrac{47.8}{100} = 0.478$

68. **(a)** P(at least some intolerance of Fascists)
$= P$(not very much) $+ P$(somewhat)
$+ P$(extremely intolerant of Facists)
$= \dfrac{20.7}{100} + \dfrac{43.1}{100} + \dfrac{22.9}{100} = \dfrac{86.7}{100} = 0.867$

(b) P(at least some intolerance of Communists)
$= P$(not very much) $+ P$(somewhat)
$+ P$(extremely intolerant of Communists)
$= \dfrac{33.0}{100} + \dfrac{34.2}{100} + \dfrac{17.1}{100} = \dfrac{84.3}{100}$
$= 0.843$

69. **(a)** There are $67 + 25 = 92$ possible judging combinations with Sasha Cohen finishing in first place. The probability of this outcome is, therefore, $92/220 = 23/55$.

(b) There are 67 possible judging combinations with Irina Slutskaya finishing in second place. The probability of this outcome is, therefore, 67/220.

(c) There are $92 + 67 = 159$ possible judging combinations with Shizuka Arahawa finishing in third place. The probability of this outcome is, therefore, 159/220.

70. The odds are as follows

$$\frac{0.03}{1-0.03} = \frac{0.03}{0.97} \text{ or 3 to 97}$$
$$\frac{0.65}{1-0.65} = \frac{0.65}{0.35} = \frac{13}{7} \text{ or 13 to 7}$$
$$\frac{0.61}{1-0.61} = \frac{0.61}{0.39} \text{ or 61 to 39}$$
$$\frac{0.21}{1-0.21} = \frac{0.21}{0.79} \text{ or 21 to 79}$$
$$\frac{0.02}{1-0.02} = \frac{0.02}{0.98} = \frac{1}{49} \text{ or 1 to 49}$$

71. The probabilities are as follows:

$$\begin{aligned} \frac{1}{1 + 195{,}199{,}999} &= \frac{1}{195{,}200{,}000} \\ &= 0.0000000051 \end{aligned}$$
$$\frac{1}{1 + 835{,}499} = \frac{1}{835{,}500} = 0.0000012$$
$$\frac{1}{1 + 157.6} = \frac{1}{158.6} = 0.0063$$
$$\frac{1}{1 + 59.32} = \frac{1}{60.32} = 0.0166$$

7.5 Conditional Probability; Independent Events

Your Turn 1

Reduce the sample space to B' and then find $n(A \cap B')$ and $n(B')$.

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{n(A \cap B')}{n(B')} = \frac{30}{55} = \frac{6}{11}$$

Your Turn 2

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$0.80 = 0.56 + 0.64 - P(E \cap F)$$
$$P(E \cap F) = 0.56 + 0.64 - 0.80$$
$$P(E \cap F) = 0.40$$
$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$
$$= \frac{0.40}{0.64} = 0.625$$

Your Turn 3

Since at least one coin is a tail, the sample space is reduced to $\{ht, th, tt\}$. Two of these equally likely outcomes have exactly one head, so $P(\text{one head}|\text{at least one tail}) = \frac{2}{3}$.

Your Turn 4

Let C represent "lives on campus" and A represent "has a car on campus." Using the given information, $P(A|C) = \frac{1}{4}$ and $P(C) = \frac{4}{5}$. By the product rule,

$$P(A \cap C) = P(A|C) \cdot P(C)$$
$$= \frac{1}{4} \cdot \frac{4}{5} = \frac{1}{5}.$$

Your Turn 5

Using Figure 23, we follow the A branch and then the U branch. Multiplying along the tree we find the probability of the composite branch, which represents $A \cap U$ or the event that A is in charge of the campaign and it produces unsatisfactory results.

$$P(A \cap U) = \frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$$

Your Turn 6

There are two paths that result in one NY plant and one Chicago plant: C first and NY second, and NY first and C second. From the tree, $P(\text{C,NY}) = \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$, and $P(\text{NY,C}) = \frac{1}{6} \cdot \frac{3}{5} = \frac{1}{10}$. So

$$P(\text{one NY plant and one Chicago plant})$$
$$= P(\text{C, NY}) + P(\text{NY, C}) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}.$$

Your Turn 7

Successive rolls of a die are independent events, so

$$P(\text{two fives in a row}) = P(\text{five}) \cdot P(\text{five})$$
$$= \frac{1}{6} \cdot \frac{1}{6}$$
$$= \frac{1}{36}.$$

Your Turn 8

Let M represent the event "you do your math homework" and H represent the event "you do your history assignment."

$$P(M) = 0.8$$
$$P(H) = 0.7$$
$$P(M \cup H) = 0.9$$

Now solve for $P(M \cap H)$.

$$P(M \cup H) = P(M) + P(H) - P(M \cap H)$$
$$0.9 = 0.8 + 0.7 - P(M \cap H)$$
$$P(M \cap H) = 0.8 + 0.7 - 0.9$$
$$P(M \cap H) = 0.6$$

However, $P(M)P(H) = (0.8)(0.7) = 0.56$, so $P(M \cap H) \neq P(M)P(H)$. Since M and H do not satisfy the product rule for independent events, they are not independent.

7.5 Exercises

1. Let A be the event "the number is 2" and B be the event "the number is odd."

The problem seeks the conditional probability $P(A|B)$. Use the definition

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Here, $P(A \cap B) = 0$ and $P(B) = \frac{1}{2}$. Thus,

$$P(A|B) = \frac{0}{\frac{1}{2}} = 0.$$

2. $$P(4|\text{even}) = \frac{P(4 \cap \text{even})}{P(\text{even})} = \frac{n(4 \cap \text{even})}{n(\text{even})} = \frac{1}{3}$$

3. Let A be the event "the number is even" and B be the event "the number is 6." Then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{6}} = 1.$$

4. Let A be the event "the number is odd" and B be the event "the number is 6." Since 6 is an even number, $P(A|B) = 0$.

5. $$P(\text{sum of 8}|\text{greater than 7}) = \frac{P(8 \cap \text{greater than 7})}{P(\text{greater than 7})} = \frac{n(8 \cap \text{greater than 7})}{n(\text{greater than 7})} = \frac{5}{15} = \frac{1}{3}$$

6. Let A be the event "sum of 6" and B be the event "double." 6 of the 36 ordered pairs are doubles, so $P(B) = \frac{6}{36} = \frac{1}{6}$. There is only one outcome, 3-3, in $A \cap B$ (that is, a double with a sum of 6), so $P(A \cap B) = \frac{1}{36}$. Thus,

$$P(A|B) = \frac{\frac{1}{36}}{\frac{1}{6}} = \frac{6}{36} = \frac{1}{6}.$$

7. The event of getting a double given that 9 was rolled is impossible; hence,

$$P(\text{double}|\text{sum of 9}) = 0.$$

8. $$P(\text{double}|\text{sum is 8}) = \frac{n(\text{double and sum is 8})}{n(\text{sum is 8})} = \frac{1}{5}$$

9. Use a reduced sample space. After the first card drawn is a heart, there remain 51 cards, of which 12 are hearts. Thus,

$$P(\text{heart on 2nd}|\text{heart on 1st}) = \frac{12}{51} = \frac{4}{17}.$$

10. $P(\text{second is black}|\text{first is a spade}) = \frac{25}{51}$, since there are 25 black cards left out of 51 cards. Note that the sample space is reduced from 52 cards to 51cards after the first card is drawn.

11. Use a reduced sample space. After the first card drawn is a jack, there remain 51 cards, of which 11 are face cards. Thus,

$$P(\text{face card on 2nd}|\text{ jack on 1st}) = \frac{11}{51}.$$

12. $P(\text{second is an ace}|\text{first is not an ace}) = \frac{4}{51}$, since there are 4 aces left out of 51 cards.

13. $$\begin{aligned} &P(\text{a jack and a 10}) \\ &= P(\text{jack followed by 10}) + P(\text{10 followed by jack}) \\ &= \frac{4}{52} \cdot \frac{4}{51} + \frac{4}{52} \cdot \frac{4}{51} \\ &= \frac{16}{2652} + \frac{16}{2652} \\ &= \frac{32}{2652} = \frac{8}{663} \end{aligned}$$

14. The probability of drawing an ace and then drawing a 4 is

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}.$$

The probability of drawing a 4 and then drawing an ace is

$$\frac{4}{52} \cdot \frac{4}{51} = \frac{4}{663}.$$

Since these are mutually exclusive, the probability of drawing an ace and a 4 is

$$\frac{4}{663} + \frac{4}{663} = \frac{8}{663}.$$

15. $$\begin{aligned} &P(\text{two black cards}) \\ &= P(\text{black on 1st}) \cdot P(\text{black on 2nd}|\text{black on 1st}) \\ &= \frac{26}{52} \cdot \frac{25}{51} \\ &= \frac{650}{2652} = \frac{25}{102} \end{aligned}$$

16. The probability that the first card is a heart is $\frac{13}{52} = \frac{1}{4}$. The probability that the second is a heart, given that the first is a heart, is $\frac{12}{51}$. Thus, the probability that both are hearts is

$$\frac{1}{4} \cdot \frac{12}{51} = \frac{1}{17}.$$

19. Examine a table of all possible outcomes of rolling a red die and rolling a green die (such as Figure 18 in Section 7.4). There are 9 outcomes of the 36 total outcomes that correspond to rolling "red die comes up even and green die comes up even"—in other words, corresponding to $A \cap B$. Therefore,

$$P(A \cap B) = \frac{9}{36} = \frac{1}{4}.$$

We also know that $P(A) = 1/2$ and $P(B) = 1/2$. Since

$$P(A \cap B) = \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = P(A) \cdot P(B),$$

the events A and B are independent.

20. The knowledge that "it rains more than 10 days" affects the knowledge that "it rains more than 15 days." (For instance, if it hasn't rained more than 10 days, it couldn't possibly have rained more than 15 days.) Since $P(D|C) \neq P(D)$, the events are dependent.

21. Notice that $P(F|E) \neq P(F)$: the knowledge that a person lives in Dallas affects the probability that the person lives in Dallas or Houston. Therefore, the events are dependent.

22. First not that $P(G) = 1/7$ and $P(H) = 1/2$. A tree diagram of all the possible outcomes of the experiment would have 14 branches, exactly one of which would correspond to the outcome "today is Tuesday and the coin comes up heads," or $G \cap H$. Therefore, $P(G \cap H) = 1/14$. But notice that

$$P(G \cap H) = \frac{1}{14} = \frac{1}{7} \cdot \frac{1}{2} = P(G) \cdot P(H).$$

Thus, the events are independent.

23. **(a)** The events that correspond to "sum is 7" are (2, 5), (3, 4), (4, 3), and (5, 2), where the first number is the number on the first slip of paper and the second number is the number on the second. Of these, only (3, 4) corresponds to "first is 3," so

$$P(\text{first is } 3|\text{sum is } 7) = \frac{1}{4}.$$

(b) The events that correspond to "sum is 8" are (3, 5) and (5, 3). Of these, only (3, 5) corresponds to "first is 3," so

$$P(\text{first is } 3|\text{sum is } 8) = \frac{1}{2}.$$

24. **(a)** Given that the first number is 5, there are four possible sums: $5 + 1 = 6$, $5 + 2 = 7$, $5 + 3 = 8$, and $5 + 4 = 9$. One of the four possible outcomes corresponds to the sum 8. Therefore,

$$P(\text{sum is } 8|\text{first number is } 5) = \frac{1}{4}.$$

(b) Given that the first number is 4, there are four possible sums: $4 + 1 = 5$, $4 + 2 = 6$, $4 + 3 = 7$, and $4 + 5 = 9$. None of these corresponds to the sum 8. Therefore,

$$P(\text{sum is } 8|\text{first number is } 4) = \frac{0}{4} = 0.$$

25. **(a)** Many answers are possible; for example, let B be the event that the first die is a 5. Then

$$P(A \cap B) = P(\text{sum is 7 and first is 5}) = \frac{1}{36}$$

$$\begin{aligned} P(A) \cdot P(B) &= P(\text{sum is 7}) \cdot P(\text{first is 5}) \\ &= \frac{6}{36} \cdot \frac{1}{6} = \frac{1}{36} \end{aligned}$$

so, $P(A \cap B) = P(A) \cdot P(B)$.

(b) Many answers are possible; for example, let B be the event that at least one die is a 5.

$$\begin{aligned} P(A \cap B) &= P(\text{sum is 7 and at least one is a 5}) \\ &= \frac{2}{36} \end{aligned}$$

$$\begin{aligned} P(A) \cdot P(B) &= P(\text{sum is 7}) \cdot P(\text{at least one is a 5}) \\ &= \frac{6}{36} \cdot \frac{11}{6} \end{aligned}$$

so, $P(A \cap B) \neq P(A) \cdot P(B)$.

29. Since A and B are independent events,

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{5} = \frac{1}{20}.$$

Thus,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{4} + \frac{1}{5} - \frac{1}{20} \\ &= \frac{2}{5}. \end{aligned}$$

30. **(a)** If A and B are mutually exclusive, then

$$\begin{aligned} P(B) &= P(A \cup B) - P(A) \\ &= 0.7 - 0.5 = 0.2 \end{aligned}$$

(b) If A and B are independent, then

$$\begin{aligned} \frac{P(A \cap B)}{P(B)} &= P(A|B) = P(A) \\ &= 0.5. \end{aligned}$$

Thus,

$$P(A \cap B) = 0.5P(B).$$

Solving

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

for

$$P(A \cap B)$$

and substituting into the previous equations we get

$$\begin{aligned} P(A) + P(B) - P(A \cup B) &= 0.5P(B) \\ 0.5 + P(B) - 0.7 &= 0.5P(B) \\ -0.2 &= -0.5P(B) \\ 0.4 &= P(B). \\ P(B) &= 0.4 \end{aligned}$$

31. At the first booth, there are three possibilities: shaker 1 has heads and shaker 2 has heads;

shaker 1 has tails and shaker 2 has heads;
shaker 1 has heads and shaker 2 has tails.
We restrict ourselves to the condition that at least one head has appeared. These three possibilities are equally likely so the probability of two heads is $\frac{1}{3}$.

At the second booth we are given the condition of one head in one shaker. The probability that the second shaker has one head is $\frac{1}{2}$.

Therefore, you stand the best chance at the second booth.

32. **(a)** Let C be the event "the coin comes up heads" and D be the event "6 is rolled on the die." We have

$$P(C) = \frac{1}{2}, \quad P(D) = \frac{1}{6},$$

$$P(C \cap D) = \frac{1}{12}.$$

So the probability of winning–of the event $C \cap D$–is 1/12.

What is the probability that either the head or the 6 occurred—in other words, what is $P(C \cup D)$? Use the union for probability.

$$\begin{aligned} P(C \cup D) &= P(C) + P(D) - P(C \cap D) \\ &= \frac{1}{2} + \frac{1}{6} - \frac{1}{12} \\ &= \frac{6}{12} + \frac{2}{12} - \frac{1}{12} \\ &= \frac{7}{12} \end{aligned}$$

We want to find the probability of "head and 6 occurred" given that "head or 6 occurred"—that is, $P((C \cap D)|(C \cup D))$. Use the formula for the conditional probability.

$$\begin{aligned} &P((C \cap D)|(C \cup D)) \\ &= \frac{P((C \cap D)|(C \cup D))}{P(C \cup D)} \\ &= \frac{P(C \cap D)}{P(C \cup D)} \\ &= \frac{\frac{1}{12}}{\frac{7}{12}} \\ &= \frac{1}{7} \end{aligned}$$

The probability of winning the original game with the additional information is now 1/7. On the other hand, the probability of winning the new game, of the event "6 is rolled," is simply $P(D) = 1/6$. Since $1/6 > 1/7$, it is better to switch.

33. No, these events are not independent.

34. For a two-child family, the sample space is

$$M - M \quad M - F \quad F - M \quad F - F.$$

$$P(\text{same sex}) = \frac{2}{4} = \frac{1}{2}$$

$$P(\text{same sex}\,|\,\text{at most one male}) = \frac{1}{3}$$

The events are not independent.

For a three-child family, the sample space is

$$\begin{array}{llll} M-M-M & M-M-F & M-F-M & M-F-F \\ F-M-M & F-M-F & F- \quad F-M & F-F-F. \end{array}$$

$$P(\text{same sex}) = \frac{2}{8} = \frac{1}{4}$$

$$P(\text{same sex}\,|\,\text{at most one male}) = \frac{1}{4}$$

The events are independent.

35. Assume that each box is equally likely to be drawn from and that within each box each marble is equally likely to be drawn. If Laura does not redistribute the marbles, then the probability of winning the Porsche is $\frac{1}{2}$, since the event of a pink marble being drawn is equivalent to the event of choosing the first of the two boxes.

If however, Laura puts 49 of the pink marbles into the second box with the 50 blue marbles, the probability of a pink marble being drawn increases to $\frac{74}{99}$. The probability of the first box being chosen is $\frac{1}{2}$, and the probability of drawing a pink marble from this box is 1. The probability of the second box being chosen is $\frac{1}{2}$, and the probability of drawing a pink marble from this box is $\frac{49}{99}$. Thus, the probability of drawing a pink marble is $\frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{49}{99} = \frac{74}{99}$. Therefore Laura increases her chances of winning by redistributing some marbles.

36. $P(C|D)$ is the probability that a customer cashes a check, given that the customer made a deposit.

$$\begin{aligned} P(C|D) &= \frac{P(C \cap D)}{P(D)} \\ &= \frac{n(C \cap D)}{n(D)} \\ &= \frac{60}{80} = \frac{3}{4} \end{aligned}$$

37. The probability that a customer cashing a check will fail to make a deposit is

$$P(D'|C) = \frac{n(D' \cap C)}{n(C)} = \frac{30}{90} = \frac{1}{3}.$$

38. $P(C'|D')$ is the probability that a customer does not cash a check, given that the customer did not make a deposit.

$$\begin{aligned} P(C'|D') &= \frac{P(C' \cap D')}{P(D')} \\ &= \frac{n(C' \cap D')}{n(D')} \\ &= \frac{10}{40} \\ &= \frac{1}{4} \end{aligned}$$

39. The probability that a customer making a deposit will not cash a check is

$$P(C'|D) = \frac{n(C' \cap D)}{n(D)} = \frac{20}{80} = \frac{1}{4}.$$

40. $P[(C \cap D)']$ is the probability that a customer does not both cash a check and make a deposit.

$$\begin{aligned} P[(C \cap D)'] &= 1 - P(C \cap D) \\ &= 1 - \frac{60}{120} \\ &= \frac{60}{120} \\ &= \frac{1}{2} \end{aligned}$$

41. **(a)** Since the separate flights are independent, the probability of 4 flights in a row is

$$(0.773)(0.773)(0.773)(0.773) \approx 0.3570$$

42. Let M represent the event "the main computer fails" and B the event "the backup computer fails." Since these events are assumed to be independent,

$$\begin{aligned} P(M \cap B) &= P(M) \cdot P(B) \\ &= 0.003(0.005) \\ &= 0.000015. \end{aligned}$$

This is the probability that both computers fail, which means that the company will not have computer service. The fraction of the time it will have service is

$$1 - 0.000015 = 0.999985.$$

Independence is a fairly realistic assumption. Situations such as floods or electric surges might cause both computers to fail at the same time.

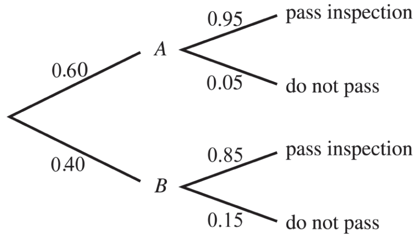
43. Let W be the event "withdraw cash from ATM" and C be the event "check account balance at ATM."

$$\begin{aligned} P(C \cup W) &= P(C) + P(W) - P(C \cap W) \\ 0.96 &= 0.32 + 0.92 - P(C \cap W) \\ -0.28 &= -P(C \cap W) \\ P(C \cap W) &= 0.28 \end{aligned}$$

$$\begin{aligned} P(W|C) &= \frac{P(C \cap W)}{P(C)} \\ &= \frac{0.28}{0.32} \\ &\approx 0.875 \end{aligned}$$

The probability that she uses an ATM to get cash given that she checked her account balance is 0.875.

Use the following tree diagram for Exercises 44 through 46.



44. Since 60% of production comes off assembly line A, $P(A) = 0.60$. Also $P(\text{pass inspection}|A) = 95$, so $P(\text{not pass}|A) = 0.05$. Therefore,

$$\begin{aligned} P(A \cap \text{not pass}) &= P(A) \cdot P(\text{not pass}|A) \\ &= 0.60(0.05) \\ &= 0.03. \end{aligned}$$

45. Since 40% of the production comes off B, $P(B) = 0.40$. Also, $P(\text{pass}|B) = 0.85$, so $P(\text{not pass}|B) = 0.15$. Therefore,

$$\begin{aligned} P(\text{not pass} \cap B) &= P(B) \cdot P(\text{not pass}|B) \\ &= 0.40(0.15) \\ &= 0.06 \end{aligned}$$

46. $P(\text{bike did not pass inspection})$

$$\begin{aligned} &= 0.60(0.05) + 0.40(0.15) \\ &= 0.03 + 0.06 \\ &= 0.09 \end{aligned}$$

47. The sample space is

$$\{RW, WR, RR, WW\}.$$

The event "red" is $\{RW, WR, RR\}$, and the event "mixed" is $\{RW, WR\}$.

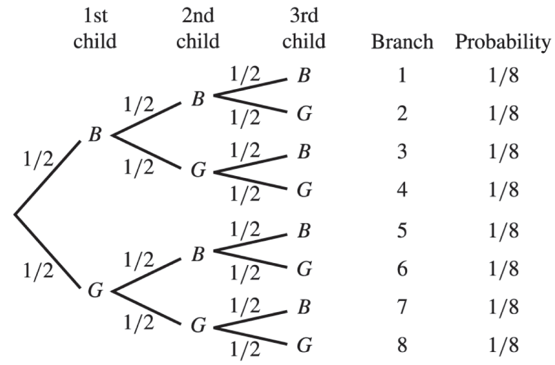
$$\begin{aligned} P(\text{mixed}|\text{red}) &= \frac{n(\text{mixed and red})}{n(\text{red})} \\ &= \frac{2}{3}. \end{aligned}$$

48. By the product rule for independent events, two events are independent if the product of their probabilities is the probability of their intersection.

$$0.75(0.4) = 0.3$$

Therefore, the given events are independent.

Use the following tree diagram for Exercises 49 through 53.



49. $P(\text{all girls}|\text{first is a girl})$

$$\begin{aligned} &= \frac{P(\text{all girls and first is a girl})}{P(\text{first is a girl})} \\ &= \frac{n(\text{all girls and first is a girl})}{n(\text{first is a girl})} \\ &= \frac{1}{4} \end{aligned}$$

50. $P(3\text{ girls}|3\text{rd is a girl})$

$$\begin{aligned} &= \frac{P(3\text{ girls and 3rd is a girl})}{P(3\text{rd is a girl})} \\ &= \frac{\frac{1}{8}}{\frac{1}{2}} \\ &= \frac{1}{4} \end{aligned}$$

51. $P(\text{all girls}|\text{second is a girl})$

$$\begin{aligned} &= \frac{P(\text{all girls and second is a girl})}{P(\text{second is a girl})} \\ &= \frac{n(\text{all girls and second is a girl})}{n(\text{second is a girl})} \\ &= \frac{1}{4} \end{aligned}$$

52. $P(3\text{ girls}\,|\,\text{at least 2 girls})$

$$\begin{aligned} &= \frac{P(3\text{ girls and at least 2 girls})}{P(\text{at least 2 girls})} \\ &= \frac{P(3\text{ girls})}{P(\text{at least 2 girls})} \\ &= \frac{P(3\text{ girls})}{P(2\text{ girls}) + P(3\text{ girls})} \\ &= \frac{\frac{1}{8}}{\frac{3}{8} + \frac{1}{8}} = \frac{\frac{1}{8}}{\frac{4}{8}} = \frac{1}{4} \end{aligned}$$

(Note that

$$P(3 \text{ girls}) = P(GGG) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$$

and

$$\begin{aligned} P(2 \text{ girls}) &= P(GGB) + P(BGG) + P(GBG) \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ &= \frac{3}{8}. \end{aligned}$$)

53. $P(\text{all girls}|\text{at least 1 girl})$

$$= \frac{P(\text{all girls and at least 1 girl})}{P(\text{at least 1 girl})}$$

$$= \frac{n(\text{all girls and at least 1 girl})}{n(\text{at least 1 girl})}$$

$$= \frac{1}{7}$$

54. $P(M) = 0.487,$ the total of the M column.

55. $P(C) = 0.039,$ the total of the C row.

56. $P(M \cap C) = 0.035,$ the entry in the M column and C row.

57. $$\begin{aligned} P(M \cup C) &= P(M) + P(C) - P(M \cap C) \\ &= 0.487 + 0.039 - 0.035 \\ &= 0.491 \end{aligned}$$

58. $$\begin{aligned} P(M|C) &= \frac{P(M \cap C)}{P(C)} \\ &= \frac{0.035}{0.039} \\ &\approx 0.897 \end{aligned}$$

59. $$\begin{aligned} P(C|M) &= \frac{P(C \cap M)}{P(M)} \\ &= \frac{0.035}{0.487} \\ &\approx 0.072 \end{aligned}$$

60. $$\begin{aligned} P(M'|C) &= \frac{P(M' \cap C)}{P(C)} \\ &= \frac{0.004}{0.039} \\ &\approx 0.103 \end{aligned}$$

61. By the definition of independent events, C and M are independent if

$$P(C|M) = P(C).$$

From Exercises 55 and 59,

$$P(C) = 0.039$$

and $$P(C|M) = 0.072.$$

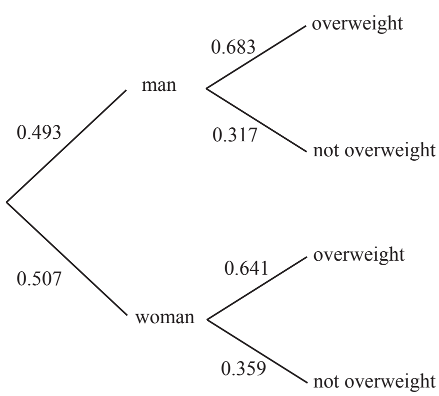
Since $P(C|M) \neq P(C),$ events C and M are not independent, so we say that they are dependent. This means that red-green color blindness does not occur equally among men and women.

62. **(a)** From the table,

$$\begin{aligned} P(C \cap D) &= 0.0008 \text{ and} \\ P(C)\cdot P(D) &= 0.0400(0.0200) \\ &= 0.0008. \end{aligned}$$

Since $P(C \cap D) = P(C)\cdot P(D),$ C and D are independent events; color blindness and deafness are independent events.

63. First draw a tree diagram.



(a) $$\begin{aligned} P(\text{overweight man}) &= (0.493)(0.683) \\ &= 0.3367 \end{aligned}$$

(b) $$\begin{aligned} P(\text{overweight}) &= 0.3367 + (0.507)(0.641) \\ &= 0.3367 + 0.3250 \\ &= 0.6617 \end{aligned}$$

(c) The two events "man" and "overweight" are independent if

$$P(\text{overweight man}) = P(\text{overweight})P(\text{man}).$$
$$P(\text{overweight})P(\text{man}) = (0.6617)(0.493) = 0.3262.$$

Since $0.3367 \neq 0.3262,$ the events are not independent

64. $$P(A) = \frac{129}{576} = \frac{43}{192}$$

65. $P(C|F) = \frac{n(C \cap F)}{n(F)} = \frac{7}{229}$

66. $P(A|H) = \frac{95}{347}$

67.
$$\begin{aligned} P(B'|H') &= P((A \cup C \cup D) \mid F) \\ &= \frac{n((A \cup C \cup D) \cap F)}{n(F)} \\ &= \frac{34 + 7 + 150}{229} \\ &= \frac{191}{229} \end{aligned}$$

68. No, $P(A) \neq P(A|H)$.

69. Let H be the event "patient has high blood pressure,"
N be the event "patient has normal blood pressure,"
L be the event "patient has low blood pressure,"
R be the event "patient has a regular heartbeat,"
and I be the event "patient has an irregular heartbeat.

We wish to determine $P(R \cap L)$.

Statement (i) tells us $P(H) = 0.14$ and statement (ii) tells us $P(L) = 0.22$. Therefore,

$$\begin{aligned} P(H) + P(N) + P(L) &= 1 \\ 0.14 + P(N) + 0.22 &= 1 \\ P(N) &= 0.64. \end{aligned}$$

Statement (iii) tells us $P(I) = 0.15$. This and statement (iv) lead to

$$P(I \cap H) = \frac{1}{3}P(I) = \frac{1}{3}(0.15) = 0.05.$$

Statement (v) tells us

$$P(N \cap I) = \frac{1}{8}P(N) = \frac{1}{3}(0.64) = 0.08.$$

Make a table and fill in the data just found.

	H	N	L	Totals
R	–	–	–	–
I	0.05	0.08	–	0.15
Totals	0.14	0.64	0.22	1.00

To determine $P(R \cap L)$, we need to find only $P(I \cap L)$.

$$\begin{aligned} P(I) &= P(I \cap H) + P(I \cap N) + P(I \cap L) \\ 0.15 &= 0.05 + 0.08 + P(I \cap L) \\ 0.15 &= 0.13 + P(I \cap L) \\ P(I \cap L) &= 0.02 \end{aligned}$$

Now calculate $P(R \cap L)$.

$$\begin{aligned} P(L) &= P(R \cap L) + P(I \cap L) \\ 0.22 &= P(R \cap L) + 0.02 \\ P(I \cap L) &= 0.20 \end{aligned}$$

The correct answer choice is **e**.

70. **(a)** $P(\text{letter is in drawer 1}) = 0.1$

(b) $P(\text{letter is in drawer 2}|\text{letter is not in drawer 1})$
$$= \frac{0.1}{0.9} \approx 0.1111$$

(c) $P(\text{letter is in drawer 3}|\text{letter is not in drawer 1 or 2})$
$$= \frac{0.1}{0.8} = 0.125$$

(d) $P(\text{letter is in drawers 8}|\text{letter is not in drawers 1 through 7})$
$$= \frac{0.1}{0.3} \approx 0.3333$$

(e) The probability is increasing.

(f) $P(\text{letter is in some drawer}) = 0.8$

(g) $P(\text{letter is in some drawer}|\text{not in drawer 1})$
$$= \frac{0.7}{0.9} \approx 0.7777$$

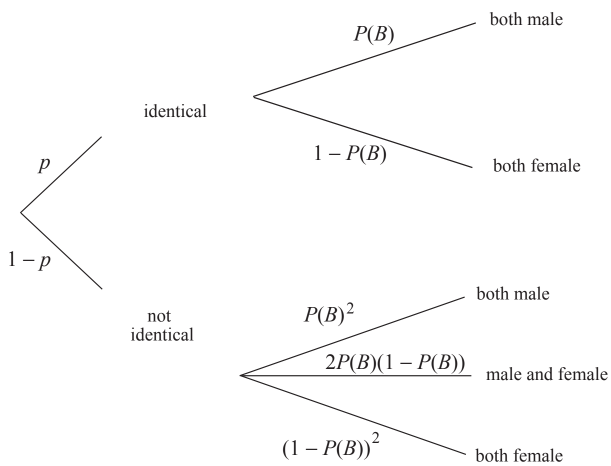
(h) $P(\text{letter is in some drawer}|\text{not in drawer 1 or 2})$
$$= \frac{0.6}{0.8} = 0.75$$

(i) $P(\text{letter is in some drawer}|\text{not in drawer 1 through 7})$
$$= \frac{0.1}{0.3} \approx 0.3333$$

(j) The probability is decreasing.

71. **(a)** The total number of males is $2(5844) + 6342 = 18{,}030$. The total number of infants is $2(17{,}798) = 35{,}596$. So among infants who are part of a twin pair, the proportion of males is $\frac{18{,}030}{35{,}596} = 0.5065$.

To answer parts (b) through (g) we make the assumption that the event "twin comes from an identical pair" is independent of the event "twin is male." We can then construct the following tree diagram.



(b) The event that the pair of twins is male happens along two compound branches. Multiplying the probabilities along these branches and adding the products we get

$$pP(B) + (1-p)(P(B))^2.$$

(c) Using the values from (a) and (b) together with the fraction of mixed twins, we solve this equation:

$$\begin{aligned}
\frac{5844}{17,798} &= p(0.506518) + (1-p)(0.506518)^2 \\
0.328352 &= 0.506518p + 0.256560 - 0.256560p \\
0.071792 &= 0.249958p \\
p &= \frac{0.071792}{0.249958} \\
p &= 0.2872
\end{aligned}$$

So our estimate for p is 0.2872. Note that if you use fewer places in the value for $P(B)$ you may get a slightly different answer.

(d) Multiplying along the two branches that result in two female twins and adding the products gives

$$p(1-P(B)) + (1-p)(1-P(B))^2.$$

(e) The equation to solve will now be the following:

$$\frac{5612}{17,798} = p(1-0.506518) + (1-p)(1-0.506518)^2$$

The answer will be the same as in part (c)

(f) Now only the "not identical" branch is involved, and the expression is

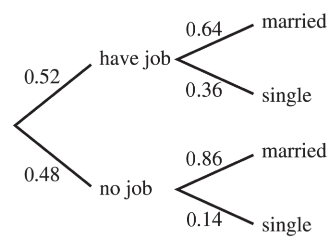
$$2(1-p)P(B)(1-P(B)).$$

(g) The equation to solve will now be the following:

$$\frac{6342}{17,798} = 2(1-p)(0.506518)(1-0.506518)$$

Again the answer will be the same as in part (c). Note that because of our independence assumption these three estimates for p must agree.

72. Draw the tree diagram.



(a)
$$\begin{aligned}
P(\text{married}) &= P(\text{job and married}) \\
&\quad + P(\text{no job and married}) \\
&= 0.52(0.64) + 0.48(0.86) \\
&= 0.3328 + 0.4128 \\
&= 0.7456
\end{aligned}$$

(b) $P(\text{job and single}) = 0.52(0.36) = 0.1872$

73. **(a)** $\dfrac{39.66}{192.64} = 0.2059$

(b) $\dfrac{28.68}{192.64} = 0.1489$

(c) $\dfrac{7.90}{192.64} = 0.0410$

(d) $\dfrac{7.90}{28.68} = 0.2755$

(e) The probability that a person is a current smoker is different from the probability that the a person is a current smoker *given* that that the person has less than a high school diploma. Thus knowing that a person has less than a high school diploma changes our estimate of the probability that the person is a smoker, so these events are not independent. Alternatively, we could note that the product of the answers to (a) and (b), which is
$P(\text{smoker}) \cdot P(\text{less than HS diploma})$

is equal to $(0.2059)(0.1489) = 0.0307$, while according to (c), P(smoker *and* less than HS diploma) $= 0.0410$. Since these values are different, the two events "current smoker" and "less than a high school diploma" are not independent.

74. **(a)**

$P(\text{forecast of rain}|\text{rain}) = \frac{66}{80} = 0.825 \approx 83\%$

$P(\text{forecast of no rain}|\text{no rain}) = \frac{764}{920} \approx 0.83 \approx 83\%$

(b) $P(\text{rain}|\text{forecast of rain}) = \frac{66}{222} \approx 0.2973$

(c) $P(\text{no rain}|\text{forecast of no rain})$
$= \frac{764}{778} \approx 0.9820$

75. **(a)** In this exercise, it is easier to work with complementary events. Let E be the event "at least one of the faults erupts." Then the complementary event E' is "none of the faults erupts," and we can use $P(E) = 1 - P(E')$.

Consider the event E': "none of the faults erupts." This means "the first fault does not erupt and the second fault does not erupt and . . . and the seventh fault does not erupt." Letting F_i denote the event "the i^{th} fault erupts," we wish to find

$$P(E') = P(F_1' \cap F_2' \cap F_3' \cap F_4' \cap F_5' \cap F_6' \cap F_7').$$

Since we are assuming the events are independent, we have

$$\begin{aligned} P(E') &= P(F_1' \cap F_2' \cap F_3' \cap F_4' \cap F_5' \cap F_6' \cap F_7') \\ &= P(F_1') \cdot P(F_2') \cdot P(F_3') \cdot P(F_4') \cdot P(F_5') \cdot P(F_6') \cdot P(F_7') \end{aligned}$$

Now use $P(F_i') = 1 - P(F_i)$ and perform the calculations.

$$\begin{aligned} P(E') &= P(F_1') \cdot P(F_2') \cdot P(F_3') \cdot P(F_4') \cdot P(F_5') \cdot P(F_6') \cdot P(F_7') \\ &= (1 - 0.27) \cdot (1 - 0.21) \cdot \ldots \cdot (1 - 0.03) \\ &= (0.73)(0.79)(0.89)(0.90)(0.96)(0.97)(0.97) \\ &\approx 0.42 \end{aligned}$$

Therefore,

$$\begin{aligned} P(E) &= 1 - P(E') \\ &\approx 1 - 0.42 \approx 0.58. \end{aligned}$$

76. $P(\text{component fails}) = 0.03$

(a) Let n represent the number of these components to be connected in parallel; that is, the component has $n - 1$ backup components.

$$\begin{aligned} &P(\text{at least one component works}) \\ &= 1 - P(\text{no component works}) \\ &= 1 - P(\text{all } n \text{ components fail}) \\ &= 1 - (0.03)^n \end{aligned}$$

If this probability is to be at least 0.999999, then

$$\begin{aligned} 1 - (0.03)^n &\geq 0.999999 \\ -(0.03)^n &\geq -0.000001 \\ (0.03)^n &\leq 0.000001, \end{aligned}$$

and the smallest whole number value of n for which this inequality holds true is $n = 4$. Therefore, $4 - 1 = 3$ backup components must be used, or 4 total components.

77. **(a)** $P(\text{second class}) = \dfrac{357}{1316} \approx 0.2713$

(b) $P(\text{surviving}) = \dfrac{499}{1316} \approx 0.3792$

(c) $P(\text{surviving}|\text{first class}) = \dfrac{203}{325} \approx 0.6246$

(d) $P(\text{surviving}|\text{child and third class}) = \dfrac{27}{79}$
≈ 0.3418

(e) $P(\text{woman}|\text{first class and survived}) = \dfrac{140}{203}$
≈ 0.6897

(f) $P(\text{third class}|\text{man and survived}) = \dfrac{75}{146}$
≈ 0.5137

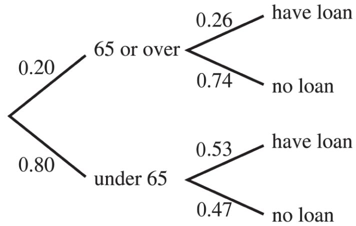
(g) $P(\text{survived}|\text{man}) = \dfrac{146}{805} \approx 0.1814$

$P(\text{survived}|\text{man and third class}) = \dfrac{75}{462}$
≈ 0.1623

No, the events are not independent.

78. The events are probably dependent, and the agent used the product rule for independent events.

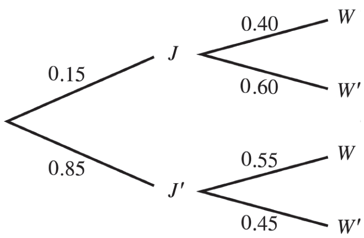
79. First draw the tree diagram.



(a) $P(\text{person is 65 or over and has a loan})$
$$= P(\text{65 or over}) \cdot P(\text{has loan} \mid \text{65 or over})$$
$$= 0.20(0.26) = 0.052$$

(b)
$$\begin{aligned} P(\text{person has a loan}) &= P(\text{65 or over and has loan}) \\ &\quad + P(\text{under 65 and has loan}) \\ &= 0.20(0.26) + 0.80(0.53) \\ &= 0.052 + 0.424 \\ &= 0.476 \end{aligned}$$

80.



From the tree diagram, we see that the probability that an individual from the community is

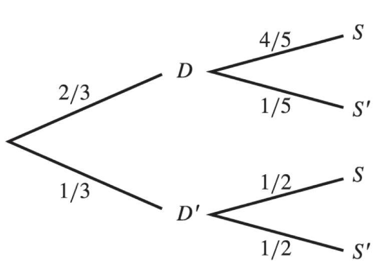
(a) a woman jogger is $0.15(0.40) = 0.060$;

(b) a man who is not a jogger is $(0.85)(0.45) = 0.3825$

(c) a woman is
$$0.15(0.40) + 0.85(0.55) = 0.06 + 0.4675 = 0.5275.$$

(d) The two events "woman" and "jogger" are independent if
$$P(\text{woman jogger}) = P(\text{woman}) \cdot P(\text{jogger}).$$
We are given that $P(\text{jogger}) = 0.15$ so that
$$P(\text{woman}) \cdot P(\text{jogger}) = 0.5275 \cdot 0.15 \approx 0.079$$
but we found $P(\text{woman jogger}) = 0.060$ in part (a). Therefore, the events are not independent.

81.

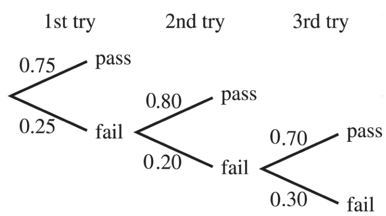


From the tree diagram, we see that the probability that a person

(a) drinks diet soft drinks is
$$\frac{2}{3}\left(\frac{4}{5}\right) + \frac{1}{3}\left(\frac{1}{2}\right) = \frac{8}{15} + \frac{1}{6} = \frac{21}{30} = \frac{7}{10};$$

(b) diets, but does not drink diet soft drinks is
$$\frac{2}{3}\left(\frac{1}{5}\right) = \frac{2}{15}.$$

82. First draw the tree diagram.



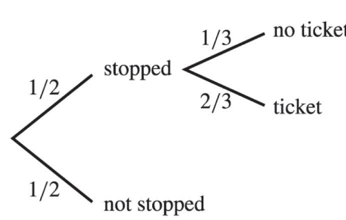
(a) $P(\text{fails both 1st and 2nd tests})$
$$= P(\text{fails 1st}) \cdot P(\text{fails 2nd} \mid \text{fails 1st})$$
$$= 0.25(0.20)$$
$$= 0.05$$

(b) $P(\text{fai1s three times in a row})$
$$= 0.25(0.20)(0.30)$$
$$= 0.015$$

(c) $P(\text{requires at least 2 tries})$
$$= P(\text{does not pass on 1st try})$$
$$= 0.25$$

83. Let F_i be the event "the ith burner fails." The event "all four burners fail" is equivalent to the event "the first burner fails <u>and</u> the second burner fails <u>and</u> the third burner fails <u>and</u> the fourth burner fails"—that is, the event $F_1 \cap F_2 \cap F_3 \cap F_4$. We are told that the burners are independent. Therefore
$$\begin{aligned} P(F_1 \cap F_2 \cap F_3 \cap F_4) &= P(F_1) \cdot P(F_2) \cdot P(F_3) \cdot P(F_4) \\ &= (0.001)(0.001)(0.001)(0.001) \\ &= 0.000000000001 = 10^{-12}. \end{aligned}$$

84. First draw a tree diagram.



(a) $P(\text{no ticket}) = \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2} = \frac{2}{3}$

(b) $P(\text{no ticket on three consecutive weekends})$
$$= \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

85. $P(\text{luxury car}) = 0.04$ and $P(\text{luxury car} \mid \text{CPA}) = 0.17$

Use the formal definition of independent events. Since these probabilities are not equal, the events are not independent.

86. Let A be the event "student studies" and B be the event "student gets a good grade." We are told that $P(A) = 0.6, P(B) = 0.7,$ and $P(A \cap B) = 0.52.$

$$P(A) \cdot P(B) = 0.6(0.7) = 0.42$$

(a) Since $P(A) \cdot P(B)$ is not equal to $P(A \cap B)$, A and B are not independent. Rather, they are dependent events.

(b) Let A be the event "a student studies" and B be the event "the student gets a good grade." Then

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{0.52}{0.6} \approx 0.87.$$

(c) Let A be the event "a student gets a good grade" and B be the event "the student studied." Then

$$P(A \mid B) = \frac{P(B \cap A)}{P(B)} = \frac{0.52}{0.7} \approx 0.74.$$

87. (a) We will assume that successive free throws are independent.

$$P(0) = 0.4$$
$$P(1) = (0.6)(0.4) = 0.24$$
$$P(2) = (0.6)(0.6) = 0.36$$

(b) Let p be her season free throw percentage.

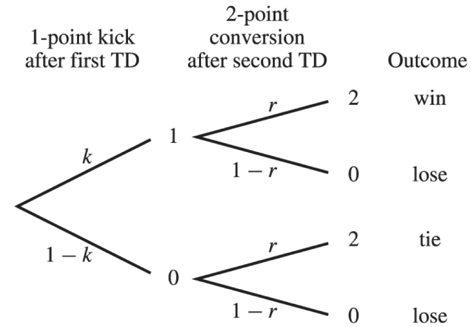
$$P(0) = 1 - p$$
$$P(2) = p^2$$

Solve:

$$p^2 = 1 - p$$
$$p^2 + p - 1 = 0$$
$$p = \frac{-1 + \sqrt{5}}{2} \approx 0.618$$

88. $P(0) = (0.4)(0.4) = 0.16$
$P(1) = 2(0.4)(0.6) = 0.48$
$P(2) = (0.6)(0.6) = 0.36$

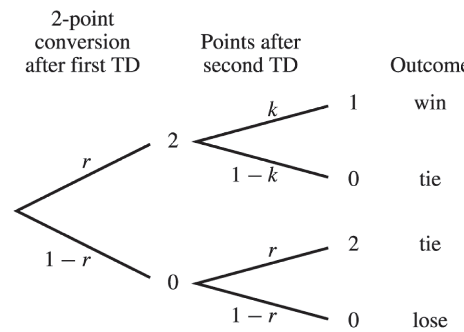
89. (a)



From the tree diagram,

$$P(\text{win}) = kr$$
$$P(\text{tie}) = (1 - k)r$$
$$P(\text{lose}) = k(1 - r) + (1 - k)(1 - r)$$
$$= k - kr + 1 - r - k + kr$$
$$= 1 - r$$

(b)



From the tree diagram,

$$P(\text{win}) = rk$$
$$P(\text{tie}) = r(1 - k) + (1 - r)r$$
$$= r - rk + r - r^2$$
$$= 2r - rk - r^2$$
$$= r(2 - k - r)$$
$$P(\text{lose}) = (1 - r)(1 - r)$$
$$= (1 - r)^2$$

(c) $P(\text{win})$ is the same under both strategies.

(d) If $r < 1, (1 - r) > (1 - r)^2.$ The probability of losing is smaller for the 2-point first strategy.

7.6 Bayes' Theorem

Your Turn 1

Let E be the event "passes math exam" and F be the event "attended review session."

$$P(E|F) = 0.8$$
$$P(E|F') = 0.65$$
$$P(F) = 0.6 \text{ so } P(F') = 0.4$$

Now apply Bayes' Theorem.

$$\begin{aligned} P(F|E) &= \frac{P(F)P(E|F)}{P(F)P(E|F) + P(F')P(E|F')} \\ &= \frac{(0.6)(0.8)}{(0.6)(0.8) + (0.4)(0.65)} \\ &= 0.6486 \end{aligned}$$

So the probability that the student attended the review session given that the student passed the exam is 0.6486.

Your Turn 2

$$\begin{aligned} \text{Let}\quad F_1 &= \text{in English I} \\ F_2 &= \text{in English II} \\ F_3 &= \text{in English III} \\ E &= \text{received help from writing center} \end{aligned}$$

Express the given information in terms of these variables.

$$\begin{aligned} &\mathrm{P}(F_1) = 0.12 \quad \mathrm{P}(E\,|\,F_1) = 0.80 \\ &\mathrm{P}(F_2) = 0.68 \quad \mathrm{P}(E\,|\,F_2) = 0.40 \\ &\mathrm{P}(F_3) = 0.20 \quad \mathrm{P}(E\,|\,F_3) = 0.11 \end{aligned}$$

$$\begin{aligned} P(F_1|E) &= \frac{P(F_1)P(E|F_1)}{P(F_1)P(E|F_1) + P(F_2)P(E|F_2) + P(F_3)P(E|F_3)} \\ &= \frac{(0.12)(0.80)}{(0.12)(0.80) + (0.68)(0.40) + (0.20)(0.11)} \\ &= 0.2462 \end{aligned}$$

Given that a student received help from the writing center, the probability that the student is in English I is 0.2462.

7.6 Exercises

1. Use Bayes' theorem with two possibilities M and M'.

$$\begin{aligned} P(M|N) &= \frac{P(M)\cdot P(N|M)}{P(M)\cdot P(N|M) + P(M')\cdot P(N|M')} \\ &= \frac{0.4(0.3)}{0.4(0.3) + 0.6(0.4)} \\ &= \frac{0.12}{0.12 + 0.24} \\ &= \frac{0.12}{0.36} = \frac{12}{36} = \frac{1}{3} \end{aligned}$$

2. By Bayes' theorem,

$$\begin{aligned}
P(M'|N) &= 1 - P(M|N) \\
&= 1 - \frac{P(M) \cdot P(N|M)}{P(M) \cdot P(N|M) + P(M') \cdot P(N|M')} \\
&= 1 - \frac{0.4(0.3)}{0.4(0.3) + 0.6(0.4)} \\
&= 1 - \frac{0.12}{0.12 + 0.24} \\
&= 1 - \frac{1}{3} = \frac{2}{3}.
\end{aligned}$$

3. Using Bayes' theorem,

$$\begin{aligned}
P(R_1|Q) &= \frac{P(R_1) \cdot P(Q|R_1)}{P(R_1) \cdot P(Q|R_1) + P(R_2) \cdot P(Q|R_2) + P(R_3) \cdot P(Q|R_3)} \\
&= \frac{0.15(0.40)}{(0.15)(0.40) + 0.55(0.20) + 0.30(0.70)} \\
&= \frac{0.06}{0.38} = \frac{6}{38} = \frac{3}{19}.
\end{aligned}$$

4. Using Bayes' theorem,

$$\begin{aligned}
P(R_2|Q) &= \frac{P(R_2) \cdot P(Q|R_2)}{P(R_1) \cdot P(Q|R_1) + P(R_2) \cdot P(Q|R_2) + P(R_3) \cdot P(Q|R_3)} \\
&= \frac{0.55(0.20)}{(0.15)(0.40) + 0.55(0.20) + 0.30(0.70)} \\
&= \frac{0.11}{0.38} = \frac{11}{38}.
\end{aligned}$$

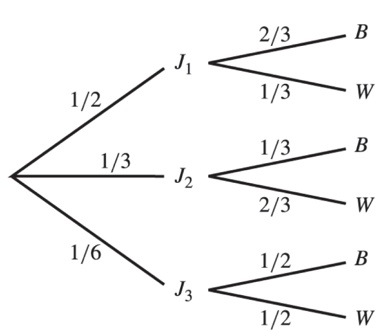
5. Using Bayes' theorem,

$$\begin{aligned}
P(R_3|Q) &= \frac{P(R_3) \cdot P(Q|R_3)}{P(R_1) \cdot P(Q|R_1) + P(R_2) \cdot P(Q|R_2) + P(R_3) \cdot P(Q|R_3)} \\
&= \frac{0.30(0.70)}{(0.15)(0.40) + 0.55(0.20) + 0.30(0.70)} \\
&= \frac{0.21}{0.38} = \frac{21}{38}.
\end{aligned}$$

6. This is the complement of the event in Exercise 3, so

$$\begin{aligned}
P(R_1'|Q) &= 1 - P(R_1|Q) \\
&= 1 - \frac{3}{19} = \frac{16}{19}.
\end{aligned}$$

7. We first draw the tree diagram and determine the probabilities as indicated below.



We want to determine the probability that if a white ball is drawn, it came from the second jar. This is $P(J_2 \mid W)$. Use Bayes' theorem.

$$P(J_2|W) = \frac{P(J_2) \cdot P(W|J_2)}{P(J_2) \cdot P(W|J_2) + P(J_1) \cdot P(W|J_1) + P(J_3) \cdot P(W|J_3)} = \frac{\frac{1}{3} \cdot \frac{2}{3}}{\frac{1}{3} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{6} \cdot \frac{1}{2}}$$

$$= \frac{\frac{2}{9}}{\frac{2}{9} + \frac{1}{6} + \frac{1}{12}} = \frac{\frac{2}{9}}{\frac{17}{36}} = \frac{8}{17}$$

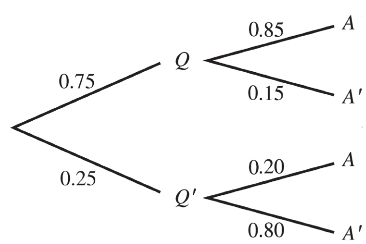
8. $$P(J_3|W) = \frac{P(J_3) \cdot P(W|J_3)}{P(J_1)P(W|J_1) + P(J_2)P(W|J_2) + P(J_3)P(W|J_3)} = \frac{\frac{1}{6} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{1}{2}} = \frac{\frac{1}{12}}{\frac{1}{6} + \frac{2}{9} + \frac{1}{12}} = \frac{3}{17}$$

9. Let G represent "good worker," B represent "bad worker," S represent "pass the test," and F represent "fail the test." The given information if $P(G) = 0.70, P(B) = P(G') = 0.30, P(S|G) = 0.85$ (and therefore $P(F|G) = 0.15$), and $P(S|B) = 0.35$ (and therefore $P(F|B) = 0.65$). If passing the test is made a requirement for employment, then the percent of the new hires that will turn out to be good workers is

$$\begin{aligned} P(G|S) &= \frac{P(G) \cdot P(S|G)}{P(G) \cdot P(S|G) + P(B) \cdot P(S|B)} \\ &= \frac{0.70(0.85)}{0.70(0.85) + 0.30(0.35)} \\ &= \frac{0.595}{0.700} \\ &= 0.85. \end{aligned}$$

85% of new hires become good workers.

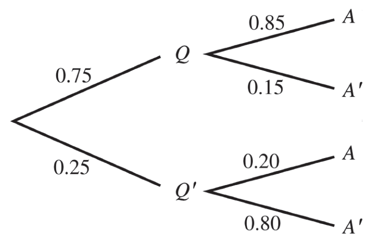
10. Let Q be the event "person is qualified" and A be the event "person was approved." Set up a tree diagram.



Using Bayes' theorem,

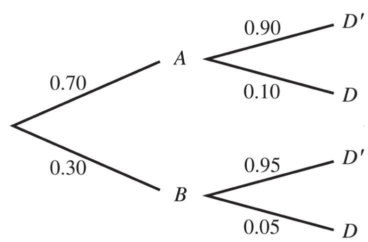
$$\begin{aligned} P(Q|A) &= \frac{P(Q) \cdot P(A|Q)}{P(Q) \cdot P(A|Q) + P(Q') \cdot P(A|Q')} \\ &= \frac{0.75(0.85)}{0.75(0.85) + 0.25(0.20)} \\ &= \frac{0.6375}{0.6875} = \frac{6375}{6875} \\ &= \frac{51}{55} \approx 0.9273 \end{aligned}$$

11. Let Q represent "qualified" and A represent "approved by the manager." Set up the tree diagram.



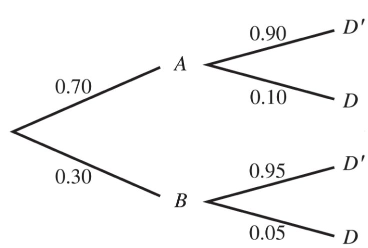
$$\begin{aligned} P(Q'|A) &= \frac{P(Q')\cdot P(A|Q')}{P(Q)\cdot P(A|Q) + P(Q')\cdot P(A|Q')} \\ &= \frac{0.25(0.20)}{0.75(0.85) + 0.25(0.20)} \\ &= \frac{0.05}{0.6875} = \frac{4}{55} \approx 0.0727 \end{aligned}$$

12. Let A be the event "the bag came from supplier A," B be the event "the bag came from supplier B," and D be the event "the bag is damaged." Set up a tree diagram.



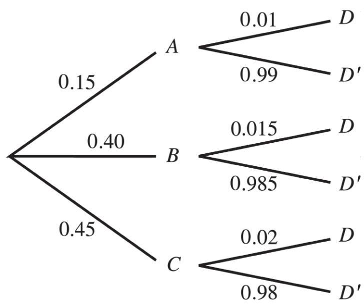
$$\begin{aligned} P(A|D) &= \frac{P(A)\cdot P(D|A)}{P(A)\cdot P(D|A) + P(B)\cdot P(D|B)} \\ &= \frac{0.70(0.10)}{0.70(0.10) + 0.30(0.05)} \\ &= \frac{0.070}{0.085} \approx 0.8235 \end{aligned}$$

13. Let D represent "damaged," A represent "from supplier A," and B represent "from supplier B." Set up the tree diagram.



$$\begin{aligned} P(B|D) &= \frac{P(B)\cdot P(D|B)}{P(B)\cdot P(D|B) + P(A)\cdot P(D|A)} \\ &= \frac{0.30(0.05)}{0.30(0.05) + 0.70(0.10)} \\ &= \frac{0.015}{0.015 + 0.07} \\ &= \frac{0.015}{0.085} \approx 0.1765 \end{aligned}$$

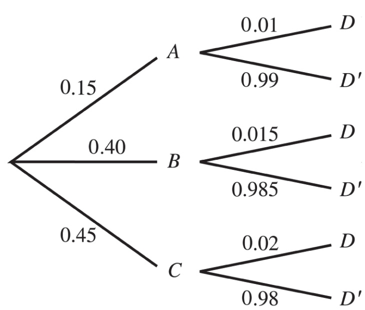
14. Let D be the event "a defective appliance" and A be the event "appliance manufactured by company A." Start with a tree diagram, where the first stage refers to the companies and the second to defective appliances.



From Bayes' theorem, we have

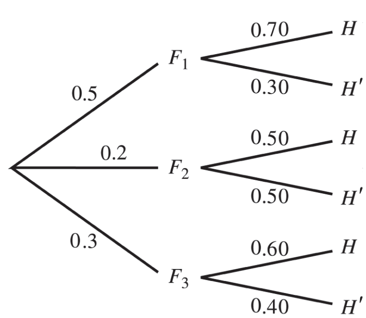
$$\begin{aligned} P(A|D) &= \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)} \\ &= \frac{0.0015}{0.0015 + 0.006 + 0.009} \\ &= \frac{0.0015}{0.0165} \approx 0.0909. \end{aligned}$$

15. Start with the tree diagram, where the first state refers to the companies and the second to a defective appliance.



$$\begin{aligned} P(B|D) &= \frac{P(B) \cdot P(D|B)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C) \cdot P(D|C)} \\ &= \frac{0.40(0.015)}{0.15(0.01) + 0.40(0.015) + 0.45(0.02)} \\ &= \frac{0.0060}{0.0165} \approx 0.3636 \end{aligned}$$

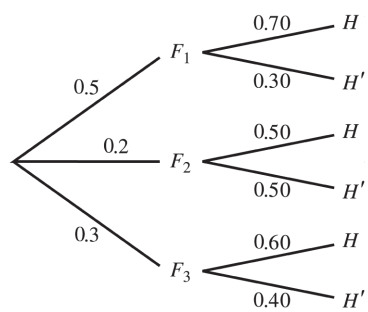
16. Let H be the event "high rating," F_1 be the event "sponsors college game," F_2 be the event "sponsors baseball game," and F_3 be the event "sponsors pro football game." First set up a tree diagram.



Now use Bayes' theorem.

$$P(F_1|H) = \frac{P(F_1) \cdot P(H|F_1)}{P(F_1)P(H|F_1) + P(F_2)P(H|F_2) + P(F_3)P(H|F_3)}$$
$$= \frac{0.5(0.70)}{0.5(0.70) + 0.2(0.50) + 0.3(0.60)} = \frac{0.35}{0.63} = \frac{5}{9}$$

17. Let H represent "high rating," F_1 represent "sponsors college game," F_2 represent "sponsors baseball game," and F_3 represent "sponsors pro football game."



$$P(F_3|H) = \frac{P(F_3) \cdot P(H|F_3)}{P(F_1) \cdot P(H|F_1) + P(F_2) \cdot P(H|F_2) + P(F_3) \cdot P(H|F_3)}$$
$$= \frac{0.3(0.60)}{0.5(0.70) + 0.2(0.50) + 0.3(0.60)}$$
$$= \frac{0.18}{0.35 + 0.10 + 0.18}$$
$$= \frac{0.18}{0.63} = \frac{18}{63} = \frac{2}{7}$$

18. Let A represent "driver is 16–20,"
B represent "driver is 21–30,"
C represent "driver is 31–65,"
D represent "driver is 66–99,"
and E represent "driver has an accident."
We wish to find $P(A|E)$.

$$P(A|E) = \frac{P(A) \cdot P(E|A)}{P(A) \cdot P(E|A) + P(B) \cdot P(E|B) + P(C) \cdot P(E|C) + P(D) \cdot P(E|D)}$$
$$= \frac{0.08(0.06)}{0.08(0.06) + 0.15(0.03) + 0.49(0.02) + 0.28(0.04)}$$
$$= \frac{0.0048}{0.0303} \approx 0.16$$

The correct choice is **b**.

19. Using the given information, construct a table similar to the one in the previous exercise.

Category of Policyholder	**Portion of Policyholders**	**Probability of Dying in the Next Year**
Standard	0.50	0.010
Preferred	0.40	0.005
Ultra-preferred	0.10	0.001

Let S represent "standard policyholder,"
R represent "preferred policyholder,"
U represent "ultra-preferred policyholder,"
and D represent "policyholder dies in the next year."
We wish to find $P(U|D)$.

$$\begin{aligned} P(U|D) &= \frac{P(U)\cdot P(D|U)}{P(S)\cdot P(D|S) + P(R)\cdot P(D|R) + P(U)\cdot P(D|U)} \\ &= \frac{0.10(0.001)}{0.50(0.010) + 0.40(0.005) + 0.10(0.001)} \\ &= \frac{0.0001}{0.0071} \approx 0.141 \end{aligned}$$

The correct answer choice is **d**.

20. Let T represent "driver is a teen,"
Y represent "driver is a young adult,"
M represent "driver is in midlife,"
S represent "driver is a senior,"
and C represent "driver has been involved in at least one collision in the past year."
We wish to find $P(Y|C)$.

$$\begin{aligned} P(Y|C) &= \frac{P(Y)\cdot P(C|Y)}{P(T)\cdot P(C|T) + P(Y)\cdot P(C|Y) + P(M)\cdot P(C|M) + P(S)\cdot P(C|S)} \\ &= \frac{0.16(0.08)}{0.08(0.15) + 0.16(0.08) + 0.45(0.04) + 0.31(0.05)} \\ &= \frac{0.0128}{0.0583} \approx 0.22 \end{aligned}$$

The correct choice is **d**.

21. Let L be the event "the object was shipped by land," A be the event "the object was shipped by air," S be the event "the object was shipped by sea," and E be the event "an error occurred."

$$\begin{aligned} P(L|E) &= \frac{P(L)\cdot P(E|L)}{P(L)\cdot P(E|L) + P(A)\cdot P(E|A) + P(S)\cdot P(E|S)} \\ &= \frac{0.50(0.02)}{0.50(0.02) + 0.40(0.04) + 0.10(0.14)} \\ &= \frac{0.0100}{0.0400} = 0.25 \end{aligned}$$

The correct response is **c**.

22. There are a total of

$$1260 + 700 + 560 + 280 = 2800$$

mortgages being studied at the bank.

(a)
$$\begin{aligned} P(5\%\text{ down}|\text{default}) &= \frac{0.05\left(\frac{1260}{2800}\right)}{0.05\left(\frac{1260}{2800}\right) + 0.03\left(\frac{700}{2800}\right) + 0.02\left(\frac{560}{2800}\right) + 0.01\left(\frac{280}{2800}\right)} \\ &= \frac{0.05(0.45)}{0.05(0.45) + 0.03(0.25) + 0.02(0.2) + 0.01(0.1)} \\ &= \frac{0.0225}{0.0225 + 0.0075 + 0.004 + 0.001} = \frac{0.0225}{0.035} \approx 0.6429 \end{aligned}$$

(b) A mortgage being paid to maturity is the complement of a mortgage being defaulted.

$$\begin{aligned} P(10\%\text{ down}|\text{paid to maturity}) &= P(10\%\text{ down}|\text{not default}) \\ &= \frac{0.97\left(\frac{700}{2800}\right)}{0.95\left(\frac{1260}{2800}\right) + 0.97\left(\frac{700}{2800}\right) + 0.98\left(\frac{560}{2800}\right) + 0.99\left(\frac{280}{2800}\right)} \\ &= \frac{0.97(0.25)}{0.95(0.45) + 0.97(0.25) + 0.98(0.2) + 0.99(0.1)} \\ &= \frac{0.2425}{0.4275 + 0.2425 + 0.196 + 0.099} = \frac{0.2425}{0.965} \approx 0.2505 \end{aligned}$$

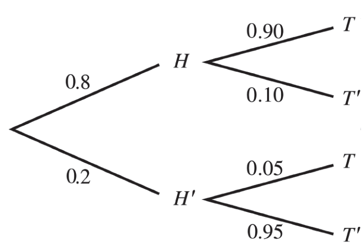
23. Let E represent the event "hemoccult test is positive," and let F represent the event "has colorectal cancer." We are given

$$P(F) = 0.003, P(E|F) = 0.5,$$
$$\text{and } P(E|F') = 0.03$$

and we want to find $P(F|E)$. Since $P(F) = 0.003, P(F') = 0.997$. Therefore,

$$P(F|E) = \frac{P(F) \cdot P(E|F)}{P(F) \cdot P(E|F) + P(F') \cdot P(E|F')} = \frac{0.003 \cdot 0.5}{0.003 \cdot 0.5 + 0.997 \cdot 0.03} \approx 0.0478.$$

24. Let H be the event "has hepatitis" and T be the event "positive test." First set up a tree diagram.



Using Bayes' theorem,

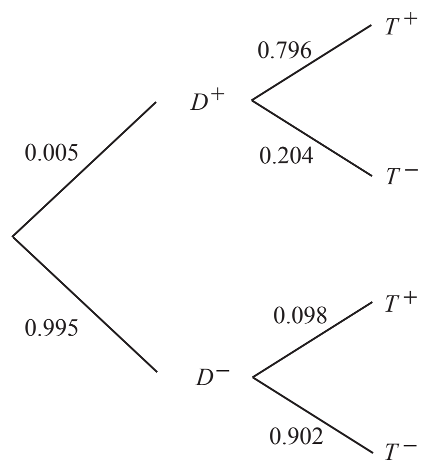
$$P(H|T) = \frac{P(H) \cdot P(T|H)}{P(H) \cdot P(T|H) + P(H') \cdot P(T|H')} = \frac{0.8(0.90)}{0.8(0.90) + 0.2(0.05)} = \frac{0.72}{0.73} = \frac{72}{73} \approx 0.9863.$$

25. $P(T^+ \mid D^+) = 0.796$

$P(T^- \mid D^-) = 0.902$

$P(D^+) = 0.005$ so $P(D^-) = 0.995$

We can now fill in the complete tree.



(a)
$$P(D^+ \mid T^+) = \frac{P(D^+)P(T^+ \mid D^+)}{P(D^+)P(T^+ \mid D^+) + P(D^-)P(T^+ \mid D^-)}$$
$$= \frac{(0.005)(0.796)}{(0.005)(0.796) + (0.995)(0.098)}$$
$$= 0.039$$

(b)
$$P(D^- \mid T^-) = \frac{P(D^-)P(T^- \mid D^-)}{P(D^-)P(T^- \mid D^-) + P(D^+)P(T^- \mid D^+)}$$
$$= \frac{(0.995)(0.902)}{(0.995)(0.902) + (0.005)(0.204)}$$
$$= 0.999$$

(c)
$$P(D^+ \mid T^-) = \frac{P(D^+)P(T^- \mid D^+)}{P(D^+)P(T^- \mid D^+) + P(D^-)P(T^- \mid D^-)}$$
$$= \frac{(0.005)(0.204)}{(0.005)(0.204) + (0.995)(0.902)}$$
$$= 0.001$$

Alternatively, since $P(D^- \mid T^-) + P(D^+ \mid T^-) = 1$, we can subtract the answer to (b) from 1.

(d) The right half of the tree stays the same, but $P(D^+)$ is now 0.015 and $P(D^-)$ is 0.985.

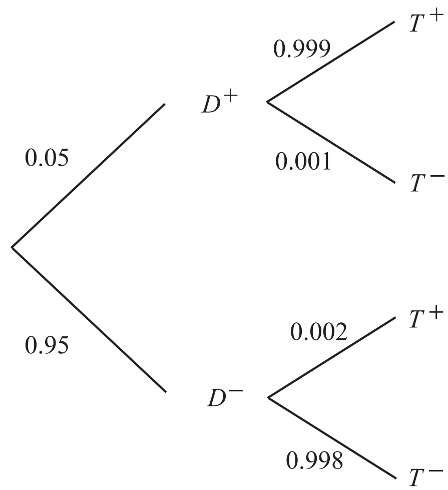
$$P(D^+ \mid T^+) = \frac{P(D^+)P(T^+ \mid D^+)}{P(D^+)P(T^+ \mid D^+) + P(D^-)P(T^+ \mid D^-)}$$
$$= \frac{(0.015)(0.796)}{(0.015)(0.796) + (0.985)(0.098)}$$
$$= 0.110$$

26. $P(T^+ \mid D^+) = 0.999$

$P(T^- \mid D^-) = 0.998$

$P(D^+) = 0.05$ so $P(D^-) = 0.95$

We can now fill in the complete tree.



(a) $$P(D^+|T^+) = \frac{P(D^+)P(T^+|D^+)}{P(D^+)P(T^+|D^+) + P(D^-)P(T^+|D^-)}$$
$$= \frac{(0.05)(0.999)}{(0.05)(0.999) + (0.95)(0.002)}$$
$$= 0.9634$$

(b) The calculation is the same except with different values for $P(D^+)$ and $P(D^-)$. Now $P(D^+) = 0.001$ and $P(D^-) = 0.999$.

$$P(D^+|T^+) = \frac{P(D^+)P(T^+|D^+)}{P(D^+)P(T^+|D^+) + P(D^-)P(T^+|D^-)}$$
$$= \frac{(0.001)(0.999)}{(0.001)(0.999) + (0.999)(0.002)}$$
$$= 0.3333$$

27. **(a)** Let H represent "heavy smoker," L be "light smoker," N be "nonsmoker," and D be "person died." Let $x = P(D|N)$, that is, let x be the probability that a nonsmoker died. Then $P(D|L) = 2x$ and $P(D|H) = 4x$. Create a table.

Level of Smoking	**Probability of Level**	**Probability of Death for Level**
H	0.2	$4x$
L	0.3	$2x$
N	0.5	x

We wish to find $P(H|D)$.

$$
\begin{aligned}
P(H|D) &= \frac{P(H) \cdot P(D|H)}{P(H) \cdot P(D|H) + P(L) \cdot P(D|L) + P(N) \cdot P(D|N)} \\
&= \frac{0.2(4x)}{0.2(4x) + 0.3(2x) + 0.5(x)} \\
&= \frac{0.8x}{1.9x} \\
&\approx 0.42
\end{aligned}
$$

The correct answer choice is **d**.

28. Let C represent "patient was critical," S be "patient was serious." T be "patient was stable," and V be "patient survived." We want to find $P(S|V)$.

The problem statement gives probabilities that a patient died in the emergency room, but we are interested in finding the probability that a patient survived. So convert the given probabilities to probabilities that a patient survived by finding the probability of the complementary event, using the result

$$P(\text{category of patient}|\text{patient survived}) = 1 - P(\text{category of patient}|\text{patient died})$$

and create a table as before.

Category of Patient	Portion of Category	Probability of Survival for Category
C	0.1	0.60
S	0.3	0.90
T	0.6	0.99

$$
\begin{aligned}
P(S|V) &= \frac{P(S) \cdot P(V|S)}{P(C) \cdot P(V|C) + P(S) \cdot P(V|S) + P(T) \cdot P(V|T)} \\
&= \frac{0.3(0.90)}{0.1(0.60) + 0.3(0.90) + 0.6(0.99)} \\
&= \frac{0.270}{0.924} \\
&\approx 0.29
\end{aligned}
$$

The correct answer choice is **b**.

29. Let H represent "person has the disease" and R be "test indicates presence of the disease." We wish to determine $P(H|R)$.

Construct a table as before.

Category of Person	Probability of Population	Probability of Presence of Disease
H	0.01	0.950
H'	0.99	0.005

$$
\begin{aligned}
P(H|R) &= \frac{P(H) \cdot P(R|H)}{P(H) \cdot P(R|H) + P(H') \cdot P(R|H')} \\
&= \frac{0.01(0.950)}{0.01(0.950) + 0.99(0.005)} \\
&= \frac{0.00950}{0.01445} \approx 0.657
\end{aligned}
$$

The correct answer choice is **b**.

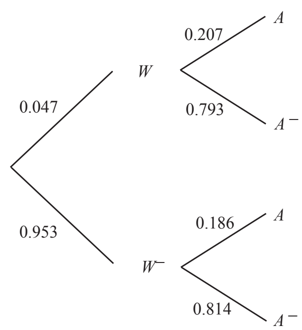
30. Let H represent "male has a circulation problem" and S be "male is a smoker." We wish to determine $P(H|S)$. Let x be the probability that the male is a smoker and construct a table as before.

Category of Male	Portion of Population	Probability of Being a Smoker
H	0.25	$2x$
H'	0.75	x

$$\begin{aligned} P(H|S) &= \frac{P(H)\cdot P(S|H)}{P(H)\cdot P(S|H) + P(H')\cdot P(S|H')} \\ &= \frac{0.25(2x)}{0.25(2x) + 0.75(x)} \\ &= \frac{0.50x}{1.25x} \approx \frac{2}{5} \end{aligned}$$

The correct answer choice is **c**.

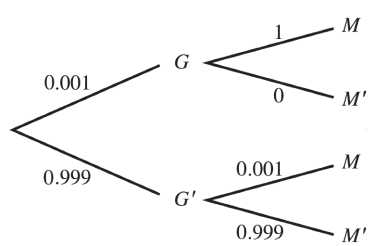
31. We start with a tree diagram based on the given information. Let W stand for "woman" and A stand for "abstains from alcohol."



(a) $$\begin{aligned} P(A) &= P(A|W)P(W) + P(A|W^-)P(W^-) \\ &= (0.047)(0.207) + (0.953)(0.186) \\ &= 0.1870 \end{aligned}$$

(b) $$\begin{aligned} P(W^- \mid A) &= \frac{P(W^-)P(A|W^-)}{P(W^-)P(A|W^-) + P(W)P(A|W)} \\ &= \frac{(0.953)(0.186)}{(0.953)(0.186) + (0.047)(0.207)} \\ &= 0.9480 \end{aligned}$$

32. Let M be the event "wife was murdered" and G be the event "husband is guilty." Set up a tree diagram.



$$P(G|M) = \frac{P(G)\cdot P(M|G)}{P(G)\cdot P(M|G) + P(G')\cdot P(M|G')} = \frac{0.001(1)}{0.001(1) + 0.999(0.001)} \approx 0.500$$

33. P(between 35 and 44 | never married) (for a randomly selected man)

$$= \frac{(0.186)(0.204)}{(0.186)(0.204) + (0.132)(0.901) + (0.186)(0.488) + (0.348)(0.118) + (0.148)(0.044)}$$
$$= 0.1285$$

34. To find the proportion of women who have been married in each age category, we subtract the numbers in the second column of the table of data for women from 1, giving the values 0.175, 0.634, 0.853, 0.908, and 0.960.

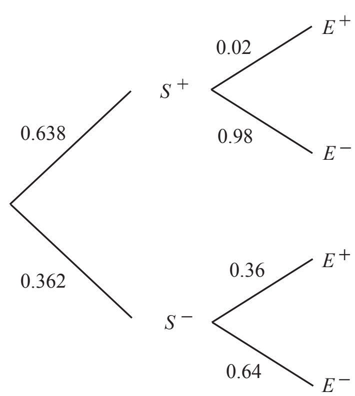
P(between 18 and 24 | has been married) (for a randomly selected woman)

$$= \frac{(0.121)(0.175)}{(0.121)(0.175) + (0.172)(0.634) + (0.178)(0.853) + (0.345)(0.908) + (0.184)(0.960)}$$
$$= 0.0274$$

35. P(between 45 and 64|never married) (for a randomly selected woman)

$$= \frac{(0.345)(0.092)}{(0.345)(0.092) + (0.121)(0.825) + (0.172)(0.366) + (0.178)(0.147) + (0.184)(0.040)}$$
$$= 0.1392$$

36. Let S^+ and S^- stand for "wore seat belt" and "did not wear seat belt" and let E^+ and E^- stand for "was ejected" and "was not ejected." We start by constructing the tree corresponding to the given data.



(a) $$P(S^+ \mid E^+) = \frac{P(S^+)P(E^+ \mid S^+)}{P(S^+)P(E^+ \mid S^+) + P(S^-)P(E^+ \mid S^-)}$$
$$= \frac{(0.638)(0.02)}{(0.638)(0.02) + (0.362)(0.36)}$$
$$= 0.0892$$

(b) $$P(S^- \mid E^-) = \frac{P(S^-)P(E^- \mid S^-)}{P(S^-)P(E^- \mid S^-) + P(S^+)P(E^- \mid S^+)}$$
$$= \frac{(0.362)(0.64)}{(0.362)(0.64) + (0.638)(0.98)}$$
$$= 0.2704$$

In Exercises 37 and 38, let S stand for "smokes" and S^{-} for "does not smoke."

37. $P(18-44|S)$

$$= \frac{P(18-44)P(S|18-44)}{P(18-44)P(S|18-44)+P(45-64)P(S|45-65)+P(65-74)P(S|65-74)+P(>75)P(S|>75)}$$

$$= \frac{(0.49)(0.23)}{(0.49)(0.23)+(0.34)(0.22)+(0.09)(0.12)+(0.08)(0.06)}$$

$$= 0.5549$$

38. $P(45-64|S^{-})$

$$= \frac{P(45-64)P(S^{-}|45-64)}{P(45-64)P(S^{-}|45-64)+P(18-44)P(S^{-}|18-44)+P(65-74)P(S^{-}|65-74)+P(>75)P(S^{-}|>75)}$$

$$= \frac{(0.34)(1-0.22)}{(0.34)(1-0.22)+(0.49)(1-0.23)+(0.09)(1-0.12)+(0.08)(1-0.06)}$$

$$= 0.3328$$

39.

Category	**Proportion of Population**	**Probability of Being Picked Up**
Has terrorist ties	$\frac{1}{1{,}000{,}000}$	0.99
Does not have terrorists ties	$\frac{999{,}999}{1{,}000{,}000}$	0.01

$$P(\text{Has terrorist ties}|\text{Picked up}) = \frac{\frac{1}{1{,}000{,}000}(0.99)}{\frac{1}{1{,}000{,}000}(0.99)+\frac{999{,}999}{1{,}000{,}000}(0.01)}$$

$$= \frac{\frac{1}{1{,}000{,}000}(0.99)}{\frac{1}{1{,}000{,}000}(0.99)+\frac{999{,}999}{1{,}000{,}000}(0.01)} \cdot \frac{1{,}000{,}000}{1{,}000{,}000}$$

$$= \frac{0.99}{10{,}000.98}$$

$$\approx 9.9 \times 10^{-5}$$

Chapter 7 Review Exercises

1. True

2. True

3. False: The union of a set with itself has the same number of elements as the set.

4. False: The intersection of a set with itself has the same number of elements as the set.

5. False: If the sets share elements, this procedure gives the wrong answer.

6. True

7. False: This procedure is correct only if the two events are mutually exclusive.

8. False: We can calculate this probability by assuming a sample space in which each card in the 52-card deck is equally likely to be drawn.

9. False: If two events A and B are mutually exclusive, then $P(A \cap B) = 0$ and this will not be equal to $P(A)P(B)$ if $P(A)$ and $P(B)$ are greater than 0.

10. True

11. False: In general these two probabilities are different. For example, for a draw from a 52-card deck, $P(\text{heart}|\text{queen}) = 1/4$ and $P(\text{queen}|\text{heart}) = 1/13$.

12. True

13. $9 \in \{8, 4, -3, -9, 6\}$

    Since 9 is not an element of the set, this statement is false.

14. $4 \notin \{3, 9, 7\}$

    Since 4 is not an element of the given set, the statement is true.

15. $2 \notin \{0, 1, 2, 3, 4\}$

    Since 2 is an element of the set, this statement is false.

16. $0 \in \{0, 1, 2, 3, 4\}$

    Since 0 is an element of the given set, the statement is true.

17. $\{3, 4, 5\} \subseteq \{2, 3, 4, 5, 6\}$

    Every element of $\{3, 4, 5\}$ is an element of $\{2, 3, 4, 5, 6\}$, so this statement is true.

18. $\{1, 2, 5, 8\} \subseteq \{1, 2, 5, 10, 11\}$

    Since 8 is an element of the first set but not of the second set, the first set cannot be a subset of the second. The statement is false.

19. $\{3, 6, 9, 10\} \subseteq \{3, 9, 11, 13\}$

    10 is an element of $\{3, 6, 9, 10\}$, but 10 is not an element of $\{3, 9, 11, 13\}$. Therefore, $\{3, 6, 9, 10\}$ is not a subset of $\{3, 9, 11, 13\}$. The statement is false.

20. $\emptyset \subseteq \{1\}$

    The empty set is a subset of every set, so the statement is true.

21. $\{2, 8\} \not\subseteq \{2, 4, 6, 8\}$

    Since both 2 and 8 are elements of $\{2, 4, 6, 8\}$, $\{2, 8\}$ is a subset of $\{2, 4, 6, 8\}$. This statement is false.

22. $0 \subseteq \emptyset$

    The empty set contains no elements and has no subsets except itself. Therefore, the statement is false.

In Exercises 23–32

$$\begin{aligned} U &= \{\text{a, b, c, d, e, f, g, h}\}, \\ K &= \{\text{c, d, e, f, h}\}, \\ \text{and } R &= \{\text{a, c, d, g}\}. \end{aligned}$$

23. K has 5 elements, so it has $2^5 = 32$ subsets.

24. $n(R) = 4,$ so R has $2^4 = 16$ subsets.

25. K' (the complement of K) is the set of all elements of U that do *not* belong to K.

    $$K' = \{\text{a, b, g}\}$$

26. $R' = \{b, e, f, h\}$ since these elements are in U but not in R.

27. $K \cap R$ (the intersection of K and R) is the set of all elements belonging to both set K and set R.

$$K \cap R = \{c, d\}$$

28. $K \cup R = \{a, c, d, e, f, g, h\}$ since these elements are in K or R, or both.

29. $(K \cap R)' = \{a, b, e, f, g, h\}$ since these elements are in U but not in $K \cap R$. (See Exercise 27.)

30. $(K \cup R)' = \{b\}$ since this element is in U but not in $K \cup R$. (See Exercise 28.)

31. $\emptyset' = U$

32. $U' = \emptyset$, which is always true.

33. $A \cap C$ is the set of all female employees in the K.O. Brown Company who are in the accounting department.

34. $B \cap D$ is the set of all sales employees who have MBA degrees.

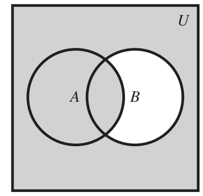
35. $A \cup D$ is the set of all employees in the K.O. Brown Company who are in the accounting department *or* have MBA degrees or both.

36. $A' \cap D$ is the set of all employees with MBA degrees who are not in the accounting department.

37. $B' \cap C'$ is the set of all male employees who are not in the sales department.

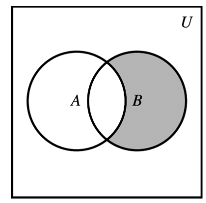
38. $(B \cup C)'$ is the set of all employees who are not either in the sales department or female, that is, all male employees not in the sales department.

39. $A \cup B'$ is the set of all elements which belong to A or do not belong to B, or both.



$A \cup B'$

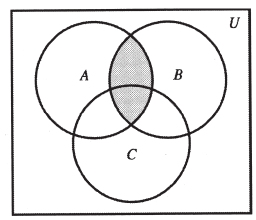
40. $A' \cap B$ contains all elements in B and not in A.



$A' \cap B$

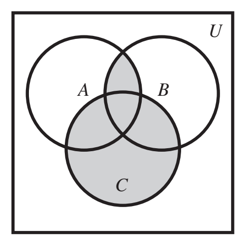
41. $(A \cap B) \cup C$

First find $A \cap B$.



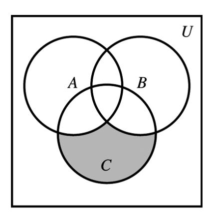
$A \cap B$

Now find the union of this region with C.



$(A \cap B) \cup C$

42. $(A \cup B)' \cap C$ includes those elements in C and not in either A or B.



$(A \cup B)' \cap C$

43. The sample space for rolling a die is

$$S = \{1, 2, 3, 4, 5, 6\}.$$

44. $S = \{\text{ace}, 2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J}, \text{Q}, \text{K}\}$

45. The sample space of the possible weights is

$$S = \{0, 0.5, 1, 1.5, 2, \ldots, 299.5, 300\}.$$

46. There are 16 possibilities.

$$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, TTHH, THTH, HTTT, THTT, TTHT, TTTH, TTTT\}$$

47. The sample space consists of all ordered pairs (a, b) where a can be 3, 5, 7, 9, or 11, and b is either R (red) or G (green). Thus,

$$S = \{(3, R), (3, G), (5, R), (5, G), (7, R), (7, G), (9, R), (9, G), (11, R), (11, G)\}.$$

48. Let R = red and G = green.

$$E = \{(7, R), (7, G), (9, R), (9, G), (11, R), (11, G)\}$$

49. The event F that the second ball is green is

$$F = \{(3, G), (5, G), (7, G), (9, G), (11, G)\}.$$

50. The outcomes are not equally likely since there are more red than green balls. For example, (7, R) is twice as likely as (7, G).

51. There are 13 hearts out of 52 cards in a deck. Thus,

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}.$$

52. There are 2 red queens out of 52 cards, so

$$P(\text{red queen}) = \frac{2}{52} = \frac{1}{26}.$$

53. There are 3 face cards in each of the four suits.

$$P(\text{face card}) = \frac{12}{52}$$

$$P(\text{heart}) = \frac{13}{52}$$

$$P(\text{face card and heart}) = \frac{3}{52}$$

$$\begin{aligned} P(\text{face card or heart}) &= P(\text{face card}) + P(\text{heart}) \\ &\quad - P(\text{face card and heart}) \\ &= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} \\ &= \frac{22}{52} = \frac{11}{26} \end{aligned}$$

54. There are 26 black cards plus 6 red face cards, so

$$P(\text{black or a face card}) = \frac{32}{52} = \frac{8}{13}.$$

55. There are 4 queens of which 2 are red, so

$$\begin{aligned} P(\text{red}\,|\,\text{queen}) &= \frac{n(\text{red and queen})}{n(\text{queen})} \\ &= \frac{2}{4} = \frac{1}{2}. \end{aligned}$$

56. There are 12 face cards, of which 4 are jacks, so

$$P(\text{jack}\,|\,\text{face card}) = \frac{4}{12} = \frac{1}{3}.$$

57. There are 4 kings of which all 4 are face cards. Thus,

$$\begin{aligned} P(\text{face card}\,|\,\text{king}) &= \frac{n(\text{face card and king})}{n(\text{king})} \\ &= \frac{4}{4} = 1. \end{aligned}$$

58. Since the king is a face card, $P(\text{king}\,|\,\text{not face card}) = 0.$

63. If A and B are nonempty and independent, then

$$P(A \cap B) = P(A) \cdot P(B).$$

For mutually exclusive events, $P(A \cap B) = 0$, which would mean $P(A) = 0$ or $P(B) = 0$. So independent events with nonzero probabilities are not mutually exclusive. But independent events one of which has zero probability are mutually exclusive.

64. Marilyn vos Savant's answer is that the contestant should switch doors. To understand why, recall that the puzzle begins with the contestant choosing door 1 and then the host opening door 3 to reveal a goat. When the host opens door 3 and shows the goat, that does not affect the probability of the car being behind door 1; the contestant had a $\frac{1}{3}$ probability of being correct to begin with, and he still has a $\frac{1}{3}$ probability after the host opens door 3.

The contestant knew that the host would open another door regardless of what was behind door 1, so opening either other door gives no new information about door 1. The probability of the car being behind door 1 is still $\frac{1}{3}$; with the goat behind door 3, the only other place the car could be is behind door 2, so the probability that the car is behind door 2 is now $\frac{2}{3}$. By switching to door 2, the contestant can double his chances of winning the car.

65. Let C be the event "a club is drawn." There are 13 clubs in the deck, so $n(C) = 13$,
$P(C') = \frac{13}{52} = \frac{1}{4}$, and $P(C') = 1 - P(C) = \frac{3}{4}$.
The odds in favor of drawing a club are

$$\frac{P(C)}{P(C')} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3},$$

which is written "1 to 3."

66. Let E represent the event "draw a black jack." $P(E) = \frac{2}{52} = \frac{1}{26}$ and then $P(E') = \frac{25}{26}$.
The odds in favor of drawing a black jack are

$$\frac{P(E)}{P(E')} = \frac{\frac{1}{26}}{\frac{25}{26}} = \frac{1}{25},$$

or 1 to 25.

67. Let R be the event "a red face card is drawn" and Q be the event "a queen is drawn." Use the union rule for probability to find $P(R \cup Q)$.

$$\begin{aligned}P(R \cup Q) &= P(R) + P(Q) - P(R \cap Q)\\ &= \frac{6}{52} + \frac{4}{52} - \frac{2}{52}\\ &= \frac{8}{52} = \frac{2}{13}\\ P(R \cup Q)' &= 1 - P(R \cup Q)\\ &= 1 - \frac{2}{13} = \frac{11}{13}\end{aligned}$$

The odds in favor of drawing a red face card or a queen are

$$\frac{P(R \cup Q)}{P(R \cup Q)'} = \frac{\frac{2}{13}}{\frac{11}{13}} = \frac{2}{11},$$

which is written "2 to 11."

68. Let E be the event "ace or club."

$$\begin{aligned}P(E) &= P(\text{ace}) + P(\text{club}) - P(\text{ace and club})\\ &= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}\\ &= \frac{16}{52} = \frac{4}{13}\end{aligned}$$

Therefore $P(E') = 1 - \frac{4}{13} = \frac{9}{13}$. The odds in favor of E are $\frac{4/13}{9/13} = \frac{4}{9}$ or 4 to 9.

69. The sum is 8 for each of the 5 outcomes 2-6, 3-5, 4-4, 5-3, and 6-2. There are 36 outcomes in all in the sample space.

$$P(\text{sum is 8}) = \frac{5}{36}$$

70. A sum of 0 is impossible, so

$$P(\text{sum is 0}) = 0.$$

71. $P(\text{sum is at least 10})$

$$\begin{aligned}&= P(\text{sum is 10}) + P(\text{sum is 11})\\ &\quad + P(\text{sum is 12})\\ &= \frac{3}{36} + \frac{2}{36} + \frac{1}{36}\\ &= \frac{6}{36} = \frac{1}{6}\end{aligned}$$

72. $P(\text{sum is no more than 5})$

$$\begin{aligned}&= P(2) + P(3) + P(4) + P(5)\\ &= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36}\\ &= \frac{10}{36} = \frac{5}{18}\end{aligned}$$

73. The sum can be 9 or 11. $P(\text{sum is 9}) = \frac{4}{36}$ and $P(\text{sum is 11}) = \frac{2}{36}$.

$$\begin{aligned}&P(\text{sum is odd number greater than 8})\\ &= \frac{4}{36} + \frac{2}{36}\\ &= \frac{6}{36} = \frac{1}{6}\end{aligned}$$

74. A roll greater than 10 means 11 or 12. There are 3 ways to get 11 or 12, 2 for 11 and 1 for 12. Hence,

$$P(12|\text{sum greater than 10}) = \frac{1}{3}.$$

75. Consider the reduced sample space of the 11 outcomes in which at least one die is a four. Of these, 2 have a sum of 7, 3-4 and 4-3. Therefore,

$$P(\text{sum is } 7 \mid \text{at least one die is a } 4) = \frac{2}{11}.$$

76. For $P(\text{at least } 9 \mid \text{one die is a } 5)$, the sample space is reduced to

$$\{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),$$
$$(1,5),(2,5),(3,5),(4,5),(6,5)\}.$$

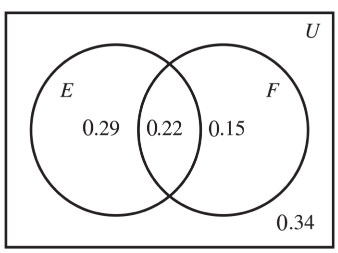
Of these 11 outcomes, 5 give a sum of 9 or more, so

$$P(\text{at least } 9 \mid \text{at least one die is } 5) = \frac{5}{11}.$$

77. $P(E) = 0.51,\ \ P(F) = 0.37,\ \ P(E \cap F) = 0.22$

(a)
$$\begin{aligned} P(E \cup F) &= P(E) + P(F) - P(E \cap F) \\ &= 0.51 + 0.37 - 0.22 \\ &= 0.66 \end{aligned}$$

(b) Draw a Venn diagram.



$E \cap F'$ is the portion of the diagram that is inside E and outside F.

$$P(E \cap F') = 0.29$$

(c) $E' \cup F$ is outside E or inside F, or both.

$$P(E' \cup F) = 0.22 + 0.15 + 0.34 = 0.71.$$

(d) $E' \cap F'$ is outside E and outside F.

$$P(E' \cap F') = 0.34$$

78. Let M represent "first urn," N represent "second urn," R represent "red ball," and B represent "blue ball." Let x be the number of blue balls in the second urn. The probability that both balls drawn are the same color is

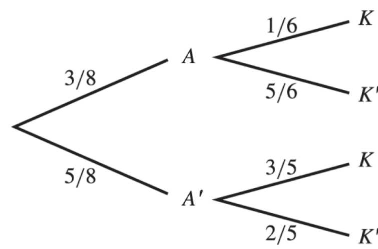
$$\begin{aligned} &P(R|M) \cdot P(R|N) + P(B|M) \cdot P(B|N) \\ &= \frac{4}{10} \cdot \frac{16}{x+16} + \frac{6}{10} \cdot \frac{x}{x+16} \\ &= \frac{6x+64}{10(x+16)} \\ &= \frac{3x+32}{5(x+16)}. \end{aligned}$$

Now set this expression equal to 0.44 and solve for x.

$$\begin{aligned} \frac{3x+32}{5(x+16)} &= 0.44 \\ 3x + 32 &= 2.2x + 35.2 \\ 0.8x &= 3.2 \\ x &= 4 \end{aligned}$$

The correct answer choice is **a**.

79. First make a tree diagram. Let A represent "box A" and K represent "black ball."



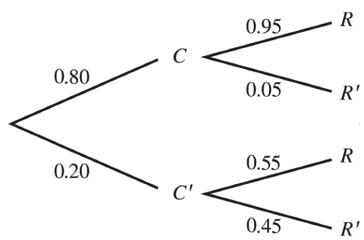
Use Bayes' theorem.

$$\begin{aligned} P(A|K) &= \frac{P(A) \cdot P(K|A)}{P(A) \cdot P(K|A) + P(A') \cdot P(K|A')} \\ &= \frac{\frac{3}{8} \cdot \frac{1}{6}}{\frac{3}{8} \cdot \frac{1}{6} + \frac{5}{8} \cdot \frac{3}{5}} \\ &= \frac{\frac{1}{16}}{\frac{7}{16}} = \frac{1}{7} \end{aligned}$$

80. The probability that the ball came from box B, given that it is red, is

$$\begin{aligned} &P(B|\text{red}) \\ &= \frac{P(B) \cdot P(\text{red}|B)}{P(B) \cdot P(\text{red}|B) + P(A) \cdot P(\text{red}|A)} \\ &= \frac{\frac{5}{8}\left(\frac{2}{5}\right)}{\frac{5}{8}\left(\frac{2}{5}\right) + \frac{3}{8}\left(\frac{5}{6}\right)} \\ &= \frac{4}{9}. \end{aligned}$$

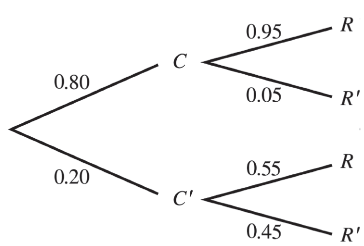
81. First make a tree diagram letting C represent "a competent shop" and R represent "an appliance is repaired correctly."



To obtain $P(C|R)$, use Bayes' theorem.

$$\begin{aligned} P(C|R) &= \frac{P(C)\cdot P(R|C)}{P(C)\cdot P(R|C) + P(C')\cdot P(R|C')} \\ &= \frac{0.80(0.95)}{0.80(0.95) + 0.20(0.55)} \\ &= \frac{0.76}{0.87} \approx 0.8736 \end{aligned}$$

82. Let C represent "competent shop" and R represent "able to repair appliance." Draw a tree diagram and label the given information.



The probability that an appliance that was repaired correctly was repaired by an incompetent shop is

$$\begin{aligned} P(C'|R) &= \frac{P(C'\cap R)}{P(R)} \\ &= \frac{P(C')\cdot P(R|C')}{P(C)\cdot P(R|C) + P(C')\cdot P(R|C')} \\ &= \frac{0.20(0.55)}{0.80(0.95) + 0.20(0.55)} \\ &= \frac{0.11}{0.76 + 0.11} = \frac{0.11}{0.87} = \frac{11}{87} \\ &\approx 0.1264. \end{aligned}$$

83. Refer to the tree diagram for Exercise 81. Use Bayes' theorem.

$$\begin{aligned} P(C|R') &= \frac{P(C)\cdot P(R'|C)}{P(C)\cdot P(R'|C) + P(C')\cdot P(R'|C')} \\ &= \frac{0.80(0.05)}{0.80(0.05) + 0.20(0.45)} \\ &= \frac{0.04}{0.13} \approx 0.3077 \end{aligned}$$

84. See the tree diagram in Exercise 81. The probability that an appliance that was repaired incorrectly was repaired by an incompetent shop is

$$\begin{aligned} P(C'|R') &= \frac{P(C'\cap R')}{P(R')} \\ &= \frac{0.20(0.45)}{0.20(0.45) + 0.80(0.05)} \\ &= \frac{0.09}{0.13} = \frac{9}{13} \approx 0.6923. \end{aligned}$$

85. To find $P(R)$, use

$$\begin{aligned} P(R) &= P(C)\cdot P(R|C) + P(C')\cdot P(R|C') \\ &= 0.80(0.95) + 0.20(0.55) = 0.87. \end{aligned}$$

86. The events C and R are independent if

$$P(C|R) = P(C).$$

From Exercise 67, $P(C|R) = 0.8736$; $P(C) = 0.80$ was given. Therefore, the events are not independent.

87. **(a)** "A customer buys neither machine" may be written $(E\cup F)'$ or $E'\cap F'$.

(b) "A customer buys at least one of the machines" is written $E\cup F$.

88. **(a)** P(no more than 3 defects)

$$\begin{aligned} &= P(0) + P(1) + P(2) + P(3) \\ &= 0.34 + 0.26 + 0.18 + 0.12 \\ &= 0.90 \end{aligned}$$

(b) P(at least 3 defects)

$$\begin{aligned} &= P(3) + P(4) + P(5) \\ &= 0.12 + 0.07 + 0.03 \\ &= 0.22 \end{aligned}$$

89. Use Bayes' theorem to find the required probabilities.

(a) Let D be the event "item is defective" and E_k be the event "item came from supplier k," $k = 1, 2, 3, 4$.

$$\begin{aligned} P(D) &= P(E_1)\cdot P(D|E_1) + P(E_2)\cdot P(D|E_2) \\ &\quad + P(E_3)\cdot P(D|E_3) + P(E_4)\cdot P(D|E_4) \\ &= 0.17(0.01) + 0.39(0.02) + 0.35(0.05) \\ &\quad + 0.09(0.03) \\ &= 0.0297 \end{aligned}$$

(b) Find $P(E_4|D)$. Using Bayes' theorem, the numerator is

$$P(E_4) \cdot P(D|E_4) = 0.09(0.03) = 0.0027.$$

The denominator is $P(E_1) \cdot P(D|E_1) + P(E_2) \cdot P(D|E_2) + P(E_3) \cdot P(D|E_3) + P(E_4) \cdot P(D|E_4)$, which, from part (a), equals 0.0297.

Therefore,

$$P(E_4|D) = \frac{0.0027}{0.0297} \approx 0.0909.$$

(c) Find $P(E_2|D)$. Using Bayes' theorem with the same denominator as in part (a),

$$\begin{aligned} P(E_2|D) &= \frac{P(E_2) \cdot P(D|E_2)}{0.0418} \\ &= \frac{0.39(0.02)}{0.0297} \\ &= \frac{0.0078}{0.0297} \\ &\approx 0.2626. \end{aligned}$$

(d) Since $P(D) = 0.0297$ and $P(D|E_4) = 0.03$,

$$P(D) \neq P(D|E_4)$$

Therefore, the events are not independent.

90. **(a)**

Car Type	Satisfied	Not Satisfied	Totals
New	300	100	400
Used	450	150	600
Totals	750	250	1000

(b) 1000 buyers were surveyed.

(c) 300 bought a new car and were satisfied.

(d) 250 were not satisfied.

(e) 600 bought used cars.

(f) 150 who were not satisfied had bought a used car.

(g) The event is "those who purchased a used car given that the buyer is not satisfied."

(h) $P(\text{used car}|\text{not satisfied})$

$$= \frac{n(\text{used car and not satisfied})}{n(\text{not satisfied})}$$

$$= \frac{150}{250} = \frac{3}{5}$$

(i) $P(\text{not satisfied}|\text{used car })$

$$= \frac{n(\text{used car and not satisfied})}{n(\text{buyers of used cars})}$$

$$= \frac{150}{600} = \frac{1}{4}$$

(k) The events "car is new" and "customer is satisfied" are independent if

$$\begin{aligned} &P(\text{car is new and customer is satisfied}) \\ &\quad = P(\text{car is new}) \cdot P(\text{customer is satisfied}). \end{aligned}$$

From the table, we have

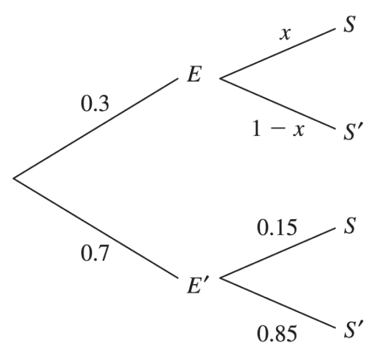
$$\begin{aligned} &P(\text{car is new}) \cdot P(\text{customer is satisfied}) \\ &\quad = \frac{400}{1000} \cdot \frac{750}{1000} = \frac{3}{10} \end{aligned}$$

and

$$\begin{aligned} &P(\text{car is new and customer is satisfied}) \\ &\quad = \frac{300}{1000} = \frac{3}{10} \end{aligned}$$

Therefore, the events are independent.

91. Let E represent "customer insures exactly one car" and S represent "customer insures a sports car." Let x be the probability that a customer who insures exactly one car insures a sports car, or $P(S|E)$. Make a tree diagram.



We are told that 20% of the customers insure a sports car, or $P(S) = 0.20$.

$$\begin{aligned} P(S) &= P(E) \cdot P(S|E) + P(E') \cdot P(S|E') \\ 0.20 &= 0.30(x) + 0.70(0.15) \\ 0.20 &= 0.3x + 0.105 \\ 0.3x &= 0.095 \\ x &\approx 0.316666667 \end{aligned}$$

Therefore, the probability that a customer insures a car other than a sports car is

$$\begin{aligned} P(S') &= 1 - P(S) \\ &\approx 1 - 0.316666667 \\ &= 0.683333333. \end{aligned}$$

Finally, the probability that a randomly selected customer insures exactly one car and that car is not a sports car is

$$\begin{aligned} P(E \cap S') &= P(E) \cdot P(S') \\ &\approx 0.3 \cdot 0.683333333 \\ &\approx 0.21. \end{aligned}$$

The correct answer choice is **b**.

92. Let Y represent the set of young policyholders, M represent the set of male policyholders, and W represent the set of married policyholders.

We want to find the number of young (Y), female (M'), and sincle (W') policyholders, or $n(Y \cap M' \cap W')$.

We are given the following information.

$n(Y) = 3000 \qquad n(M) = 4600 \qquad n(W) = 7000$

$n(Y \cap M) = 1320$

$n(M \cap W) = 3010$

$n(Y \cap M) = 1400$

$n(Y \cap M \cap M) = 600$

Use a Venn diagram and enter the information as it is found.

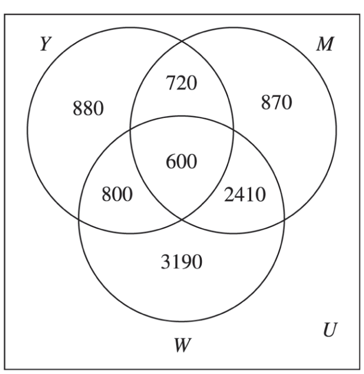
Begin with $n(Y \cap M \cap W) = 600$.
Since $n(Y \cap M) = 1400$, the number in $Y \cap W$ but not in M, or $n(Y \cap M' \cap W)$, is $1400 - 600 = 800$.

Since $n(Y \cap M) = 1320$,

$$n(Y \cap M \cap W') = 1320 - 600 = 720.$$

Since

$$\begin{aligned} n(Y) &= 3000, \\ n(Y \cap M' \cap W') &= 3000 - 600 - 720 - 800 \\ &= 880. \end{aligned}$$



So the correct answer choice is **d**.

93. Let C represent "the automobile owner purchases collision coverage" and D represent "the automobile owner purchases disability coverage." We want to find $P(C' \cap D') = P[(C \cup D)'] = 1 - P(C \cup D)$. We are given that $P(C) = 2 \cdot P(D)$ and that $P(C \cap D) = 0.15$. Let $x = P(D)$.

$$\begin{aligned} P(C \cap D) &= P(C) \cdot P(D) \\ 0.15 &= 2x \cdot x \\ 0.075 &= x^2 \\ x &= \sqrt{0.075} \\ x &\approx 0.2739 \end{aligned}$$

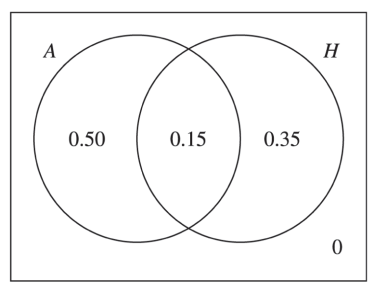
So $P(D) = x \approx 0.2739$ and $P(C) = 2x$.

$$\begin{aligned} P(C \cup D) &= P(C) + P(D) - P(C \cap D) \\ &= 2(0.2739) + 0.2739 - 0.15 \\ &= 0.6720 \end{aligned}$$

$$\begin{aligned} P(C' \cap D') &= 1 - P(C \cup D) \\ &= 1 - 0.6720 \\ &\approx 0.33 \end{aligned}$$

The correct choice is answer **b**.

94. Let A represent "the policyholder has auto policy H represent "the policyholder has homeowners policy" and R represent "the policyholder will renew." We want to find $P(R|A \cup H)$, the probability, or percentage, of policyholders who will renew at least one policy. We are given $P(A) = 0.65$, $P(H) = 0.50$, and $P(A \cap H) = 0.15$. Complete a Venn diagram.



We are also told "40% of policyholders who have only an auto policy will renew," so $P(R|A \cap H') = 0.40$.
We are similarly given $P(R|H) = 60$ and $P(R|A \cap H) = 0.80$.
Using the Venn diagram as a guide, we have

$$\begin{aligned} P(R|A \cup H) &= P(A \cap H')P(R|A \cap H') \\ &\quad + P(H \cap A')P(R|H \cap A') \\ &\quad + P(A \cap H)P(R|A \cap H) \\ &= 0.50(0.40) + 0.35(0.60) + 0.15(0.80) \\ &= 0.53 \end{aligned}$$

The correct answer choice is **d**.

95. **(a)**

	N_2	T_2
N_1	N_1N_2	N_1T_2
T_1	T_1N_2	T_1T_2

Since the four combinations are equally likely, each has probability $\frac{1}{2}$.

(b) $P(\text{two trait cells}) = P(T_1T_2) = \frac{1}{4}$

(c) $P(\text{one normal cell and one trait cell})$

$$= P(N_1T_2) + P(T_1N_2)$$
$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(d) $P(\text{not a carrier and does not have disease})$

$$= P(N_1N_2) = \frac{1}{4}$$

96. Let $P(E)$ be the probability the random donor has blood type E.

(a) $P(O^+) + P(O^-)$

$$= 0.38 + 0.08$$
$$= 0.46, \text{ or } 46\%$$

(b) $P(A^+) + P(A^-) + P(O^+) + P(O^-)$

$$= 0.32 + 0.07 + 0.38 + 0.08$$
$$= 0.85. \text{ or } 85\%$$

(c) $P(B^+) + P(B^-) + P(O^+) + P(O^-)$

$$= 0.09 + 0.02 + 0.38 + 0.08$$
$$= 0.57, \text{ or } 57\%$$

(d) $P(A^-) + P(O^-)$

$$= 0.07 + 0.08$$
$$= 0.15, \text{ or } 15\%$$

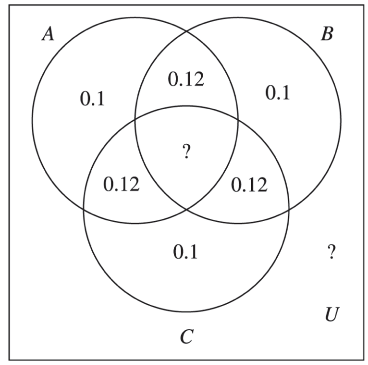
(e) $P(B^-) + P(O^-)$

$$= 0.02 + 0.08$$
$$= 0.10, \text{ or } 10\%$$

(f) $P(AB^-) + P(A^-) + P(B^-) + P(O^-)$

$$= 0.01 + 0.07 + 0.02 + 0.08$$
$$= 0.18, \text{ or } 18\%$$

97. We want to find $P(A' \cap B' \cap C' | A')$. Use a Venn diagram, fill in the information given, and use the diagram to help determine the missing values.



To determine $P(A \cap B \cap C)$, we are told that "The probability that a woman has all three risk factors, given that she has A and B, is 1/3." Therefore, $P(A \cap B \cap C \mid A \cap B) = 1/3$.
Let $x = P(A \cap B)$; then, using the diagram as a guide,

$$P(A \cap B \cap C) + P(A \cap B \cap C') = P(A \cap B)$$
$$\frac{1}{3}x + 0.12 = x$$
$$0.12 = \frac{2}{3}x$$
$$x = 0.18$$

So, $P(A \cap B \cap C) = (1/3)(0.16) = 0.06$. By DeMorgan's laws, we have

$$A' \cap B' \cap C' = (A \cup B \cup C)'$$

so that

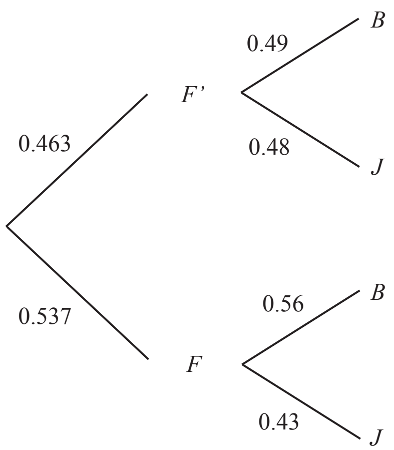
$$\begin{aligned} P(A' \cap B' \cap C') &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - [3(0.10) + 3(0.12) + 0.06] \\ &= 0.28. \end{aligned}$$

Therefore,

$$\begin{aligned} P(A' \cap B' \cap C' | A') &= \frac{P(A' \cap B' \cap C' \cap A')}{P(A')} \\ &= \frac{P(A' \cap B' \cap C')}{P(A')} \\ &= \frac{0.28}{0.6} \approx 0.467. \end{aligned}$$

The correct answer choice is **c**.

98. Let F represent "female," B represent "voted for Obama" and J represent "voted for McCain." The tree corresponding to the given data looks like this:



(We could add a third branch to each pair on the right to represent the women and men who voted for neither Obama nor McCain, but we won't need this.)

(a) The fraction of voters who voted for Obama is

$$P(F)P(B|F) + P(F')P(B|F')$$
$$= (0.537)(0.56) + (0.463)(0.49)$$
$$= 0.5276.$$

(b) $$P(F'|B) = \frac{P(F')P(B|F')}{P(F')P(B|F') + P(F)P(B|F)} = \frac{(0.463)(0.49)}{0.5276} = 0.4300$$

(Note that we already computed the denominator we need in part (a).)

(c) $P(F|B)$ is just 1 minus the value we found in (b), or 0.5700.

99. Let C be the set of viewers who watch situation comedies,
G be the set of viewers who watch game shows,
and M be the set of viewers who watch movies.

We are given the following information.

$n(C) = 20 \quad n(G) = 19 \quad n(M) = 27$
$n(M \cap G') = 19$
$n(C \cap G') = 15$
$n(C \cap M) = 10$
$n(C \cap G \cap M) = 3$
$n(C' \cap G' \cap M') = 7$

Start with $C \cap G \cap M$: $n(C \cap G \cap M) = 3$.

Since $n(C \cap M) = 10$, the number of people who watched comedies and movies but not game shows, or $n(C \cap G' \cap M)$, is $10 - 3 = 7$.

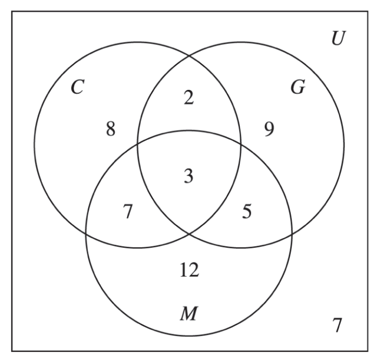
Since $n(M \cap G') = 19$, $n(C' \cap G' \cap M) = 19 - 7 = 12$.

Since $n(M) = 27$, $n(C' \cap G \cap M) = 27 - 3 - 7 - 12 = 5$.

Since $n(C \cap G') = 15$, $n(C \cap G' \cap M') = 15 - 7 = 8$.

Since $n(C) = 20$,
$n(C \cap G \cap M') = 20 - 8 - 3 - 7 = 2$.

Finally, since $n(G) = 19$, $n(C' \cap G \cap M') = 19 - 2 - 3 - 5 = 9$.



(a) $n(U) = 8 + 2 + 9 + 7 + 3 + 5 + 12 + 7 = 53$

(b) $n(C \cap G' \cap M) = 7$

(c) $n(C' \cap G' \cap M) = 12$

(d) $n(M') = n(U) - n(M) = 53 - 27 = 26$

100. **(a)** $P(\text{answer yes}) = P(\text{question } B) \cdot P(\text{answer yes}|\text{answer } B) + P(\text{answer } A) \cdot P(\text{answer yes}|\text{question } A)$

Divide by $P(\text{question } B)$.

$$\frac{P(\text{answer yes})}{P(\text{question } B)} = P(\text{answer yes}|\text{question } B) + \frac{P(\text{question } A) \cdot P(\text{answer yes}|\text{question } A)}{P(\text{question } B)}$$

Solve for $P(\text{answer yes}|\text{question } B)$.

$$P(\text{answer yes} \mid \text{question } B) = \frac{P(\text{answer yes}) - P(\text{question } A) \cdot P(\text{answer yes}|\text{question } A)}{P(\text{question } B)}$$

(b) Using the formula from part (a),

$$\frac{0.6 - \frac{1}{2}\left(\frac{1}{2}\right)}{\frac{1}{2}} = \frac{7}{10}.$$

101. Let C be the event "the culprit penny is chosen." Then

$$p(C|HHH) = \frac{P(C \cap HHH)}{P(HHH)}.$$

These heads will result two different ways. The culprit coin is chosen $\frac{1}{3}$ of the time and the probability of a head on any one flip is $\frac{3}{4}$: $P(C \cap HHH) = \frac{1}{3}\left(\frac{3}{4}\right)^3 \approx 0.1406$. If a fair (innocent) coin is chosen, the probability of a head on any one flip is $\frac{1}{2}$: $P(C'|HHH) = \frac{2}{3}\left(\frac{1}{2}\right)^3 \approx 0.0833$. Therefore,

$$\begin{aligned} P(C|HHH) &= \frac{P(C \cap HHH)}{P(HHH)} \\ &= \frac{P(C \cap HHH)}{P(C \cap HHH) + P(C' \cap HHH)} \\ &\approx \frac{0.1406}{0.1406 + 0.0833} \\ &\approx 0.6279 \end{aligned}$$

102. In calculating the probability that two babies in a family would die of SIDS is $(1/8543)^2$, he assumed that the events that either infant died of SIDS are independent. There may be a genetic factor, in which case the events are dependent.

103. $P(\text{earthquake}) = \dfrac{9}{9+1} = \dfrac{9}{10} = 0.90$

104. **(a)** $P(\text{making a 1st down with } n \text{ yards to go}) = \dfrac{\text{number of successes}}{\text{number of trials}}$

n	Trials	Successes	Probability of Making First Down with n Yards to Go
1	543	388	$\frac{388}{543} \approx 0.7145$
2	327	186	$\frac{186}{327} \approx 0.5688$
3	356	146	$\frac{146}{356} \approx 0.4101$
4	302	97	$\frac{97}{302} \approx 0.3212$
5	336	91	$\frac{91}{336} \approx 0.2708$

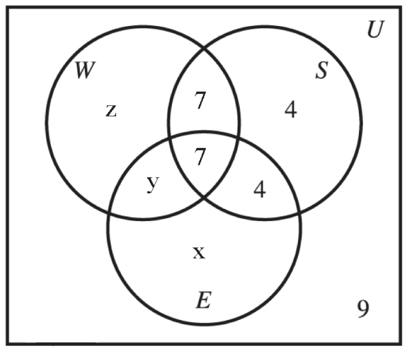
105. Let W be the set of western states,
S be the set of small states, and
E be the set of early states.

We are given the following information.

$n(W) = 24$ $\quad n(S) = 22$ $\quad n(E) = 26$

$n(W' \cap S' \cap E') = 9$ $\quad n(W \cap S) = 14$

$n(S \cap E) = 11$ $\quad n(W \cap S \cap E) = 7$



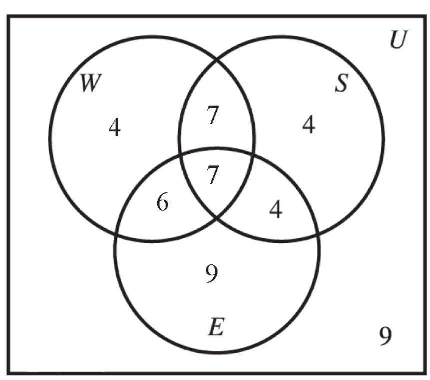
First, put 7 in $W \cap S \cap E$ and 9 in $W' \cap S' \cap E'$.
Complete $S \cap E$ with 4 for a total of 11.
Complete $W \cap S$ with 7 for a total of 14.
Complete S with 4 for a total of 22.

To complete the rest of the diagram requires solving some equations. Let the incomplete region of E be x, the incomplete region of $W \cap E$ be y, and the incomplete region of W be z. Then, using the given values and the fact that $n\,(U) = 50$,

$$\begin{aligned} x + y \quad\;\; &= 15 \\ y + z &= 10 \\ x + y + z &= 50 - 22 - 9 = 19. \end{aligned}$$

The solution to the system is $x = 9,\ y = 6,$ and $z = 4$.

Complete $W \cap E$ with.
Complete E with 9 for a total of 26.
Complete W with 4 for a total of 24.
The completed diagram is as follows.



(a) $n(W \cap S' \cap E') = 4$

(b) $n(W' \cap S') = n((W \cup S)') = 18$

106. Let L be the set of songs about love,
P be the set of songs about prison,
and T be the set of songs about trucks.

We are given the following information.

$n(L \cap P \cap T) = 12$ $\quad n(L \cap P) = 13$ $\quad n(L) = 28$ $\quad n(L \cap T) = 18$

$n(L' \cap P) = 5$ $\quad n(P) = 18$ $\quad n(P \cap T) = 15$ $\quad n(P' \cap T) = 16$

Start with $L \cap P \cap T$: $n(L \cap P \cap T) = 12$.

Since $n(L \cap P) = 13$, $n(L \cap P \cap T') = 1$.

Since $n(L \cap T) = 18$, $n(L \cap P' \cap T) = 6$.

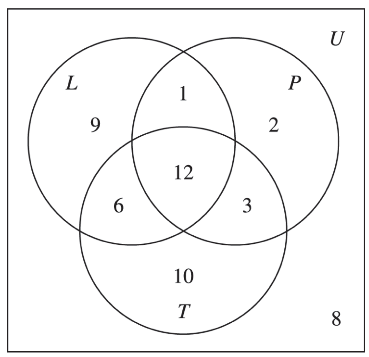
Since $n(L) = 28$, $n(L \cap P' \cap T') = 28 - 12 - 1 - 6 = 9$.

Since $n(P \cap T) = 15$, $n(L' \cap P \cap T) = 3$.

Since $n(P) = 18$, $n(L' \cap P \cap T') = 18 - 1 - 12 - 3 = 2$.

Since $n(P' \cap T) = 16$, $n(L' \cap P' \cap T) = 16 - 6 = 10$.

Finally, $n(L' \cap P' \cap T') = 8$.



(a) $n(U) = 9 + 1 + 2 + 6 + 12 + 3 + 10 + 8 = 51$

(b) $n(T) = 6 + 12 + 3 + 10 = 31$

(c) $n(P \cap T' \cap L') = 2$

(d) $n(P \cap T' \cap L) = 1$

(e) $n(P \cap T') = 1 + 2 = 3$

(f) $n(L') = n(U) - n(L) = 51 - 28 = 23$

107. Let R be "a red side is facing up" and
RR be "the 2-sided red card is chosen."

If a red side is facing up, we want to find $P(RR|R)$ since the other possibility would be a green side is facing down.

$$P(RR|R) = \frac{P(RR)}{P(R)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

No, the bet is not a good bet.

108. **(a)** $P(\text{double miss}) = 0.05(0.05) = 0.0025$

(b) $P(\text{specific silo destroyed}) = 1 - P(\text{double miss}) = 1 - 0.0025 = 0.9975$

(c) $P(\text{all ten destroyed}) = (0.9975)^{10} \approx 0.9753$

(d) $P(\text{at least one survived})$

$$= 1 - P(\text{none survived}) = 1 - P(\text{all ten destroyed})$$
$$= 1 - 0.9753 = 0.0247 \text{ or } 2.47\%$$

This does not agree with the quote of a 5% chance that at least one would survive.

(e) The events that each of the two bombs hit their targets are assumed to be independent. The events that each silo is destroyed are assumed to be independent.

109. Let G be the set of people who watched gymnastics,
B be the set of people who watched baseball,
and S be the set of people who watched soccer.

We want to find $P(G' \cap B' \cap S')$ or, by DeMorgan's laws, $P[(G \cup B \cup S)']$.

We are given the following information.

$P(G) = 0.28$ $\quad P(B) = 0.29$ $\quad P(S) = 0.19$ $\quad P(G \cap B) = 0.14$
$P(B \cap S) = 0.12$ $\quad P(G \cap S) = 0.10$ $\quad P(G \cap B \cap S) = 0.08$

Start with $P(G \cap B \cap S) = 0.08$ and work from the inside out.

Since $P(G \cap S) = 0.10$, $P(G \cap B' \cap S) = 0.02$.

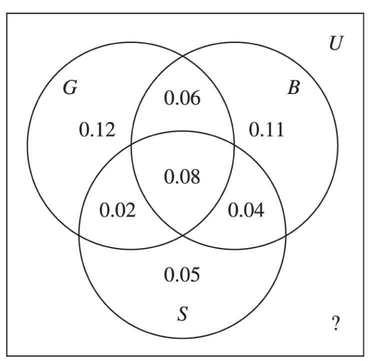
Since $P(B \cap S) = 0.12$, $P(G' \cap B \cap S) = 0.04$.

Since $P(G \cap B) = 0.14$, $P(G \cap B \cap S') = 0.06$.

Since $P(S) = 0.19, P(G' \cap B' \cap S) = 0.19 - 0.14 = 0.05$.

Since $P(B) = 0.29, P(G' \cap B \cap S') = 0.29 - 0.18 = 0.11$.

Since $P(G) = 0.28, P(G \cap B' \cap S') = 0.28 - 0.16 = 0.12$.



Therfore,

$$P(G' \cap B' \cap S) = P[(G \cup B \cup S)']$$
$$= 1 - P(G \cup B \cup S)$$
$$= 1 - (0.12 + 0.06 + 0.11 + 0.02 + 0.08 + 0.04 + 0.05) = 0.52.$$
$$P(G' \cap B' \cap S) = 1 - (0.12 + 0.06 + 0.11 + 0.02 + 0.08 + 0.04 + 0.05) = 0.52.$$

The correct answer choice is **d**.

Extended Application: Medical Diagnosis

1. Using Bayes' theorem,

$$\begin{aligned}P(H_2|C_1) &= \frac{P(C_1|H_2)\cdot P(H_2)}{P(C_1|H_1)P(H_1)+P(C_1|H_2)P(H_2)+P(C_1|H_3)P(H_3)}\\ &= \frac{0.4(0.15)}{0.9(0.8)+0.4(0.15)+0.1(0.05)}\\ &= \frac{0.06}{0.785}\approx 0.076.\end{aligned}$$

2. Using Bayes' theorem,

$$\begin{aligned}P(H_1|C_2) &= \frac{P(C_2|H_1)\cdot P(H_1)}{P(C_2|H_1)P(H_1)+P(C_2|H_2)P(H_2)+P(C_2|H_3)P(H_3)}\\ &= \frac{0.2(0.8)}{0.2(0.8)+0.8(0.15)+0.3(0.05)}\\ &= \frac{0.16}{0.295}\approx 0.542.\end{aligned}$$

3. Using Bayes' theorem,

$$\begin{aligned}P(H_3|C_2) &= \frac{P(C_2|H_3)\cdot P(H_3)}{P(C_2|H_1)P(H_1)+P(C_2|H_2)P(H_2)+P(C_2|H_3)P(H_3)}\\ &= \frac{0.3(0.05)}{0.2(0.8)+0.8(0.15)+0.3(0.05)}\\ &= \frac{0.015}{0.295}\approx 0.051.\end{aligned}$$